Professional Engineers Ontario

National Exams – May 2012

07-Str-B3

Applications of Finite Elements

3 hours duration

Notes:

1. There are 4 pages in this examination.

2. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made

3. This is a closed book examination.

4. Candidates may use one of the approved Casio or Sharp calculators

5. **Answer only TWO (2) problems out of the three proposed.** The first two problems as they appear in the answer book will be marked.

6. All problems are of equal value.

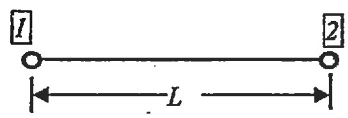
Problem 1:

1. Explain the source of errors in finite element modelling.

2. The solution of an elasticity problem obtained by the Finite Element Method does not guaranty equilibrium within the domain, explain why?

3. Give the characteristics of the shape functions needed for the Bernoulli's beam formulation.

4. Draw the approximate shape functions for the Bernoulli's **beam element** shown in the following figure.

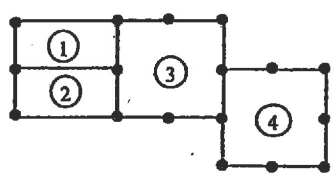


Beam element

5. Explain the relationship between the number of integration points and the order of a 2D elasticity finite element.

6. The interpolation functions for the four-nodes quadrilateral element verify the following condition: $\sum_{i=1}^{4} N_i(\xi,\eta) = 1$ for any couple (ξ,η), what is the significance of this observation?

7. Identify the defects associated with connecting four-node and eight-node elements in the pattern shown



8. Explain why linear quadrilateral elements are not good candidates for flexural dominant two-dimensional elasticity problems?

9. Finite element analysis gives stresses in general coordinate directions in terms of σ_x, σ_y, etc. Discuss how you can interpret these results for ductile (e.g. mild steel) and brittle (e.g. concrete) materials.

10. What happens if the Poisson's ratio, ν, becomes close to 0.5 in the plane strain case? What is the implication in modeling such materials using the finite element method?

Problem 2

2.1 Show that the interpolation of the displacement field inside the rectangular element, illustrated in Fig 2(a), is defined by $u(x,y)=\sum_{i=1}^{4} N_i(x,y)\,u_i$, where N_i are given by:

$$N_1(x,y)=\frac{(a-x)(b-y)}{4ab} \qquad N_3(x,y)=\frac{(a+x)(b+y)}{4ab}$$

$$N_2(x,y)=\frac{(a+x)(b-y)}{4ab} \qquad N_4(x,y)=\frac{(a-x)(b+y)}{4ab}$$

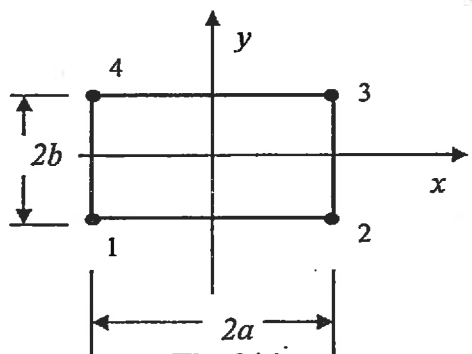


Fig. 2(a)

2.2 The above illustrated four nodes rectangular element is made from an elastic material with an elasticity modulus E and a Poisson's ratio v and has a thickness t. Show that in the case of plane stress, the stiffness term k_{33} is given by:

$$k_{33}=\left(\frac{b}{3a}+\frac{1-v}{6}\frac{a}{b}\right)\frac{Et}{1-v^2}$$

2.3 Figure 2(b) shows a plate of thickness t reinforced at its center by a bar with constant cross section A. The plate and the bar are made from the same material which has an elastic modulus equal to E. The applied force P is also illustrated in the figure.
Use $E=30\times10^6\ N/cm^2$ and $v=0.3$.

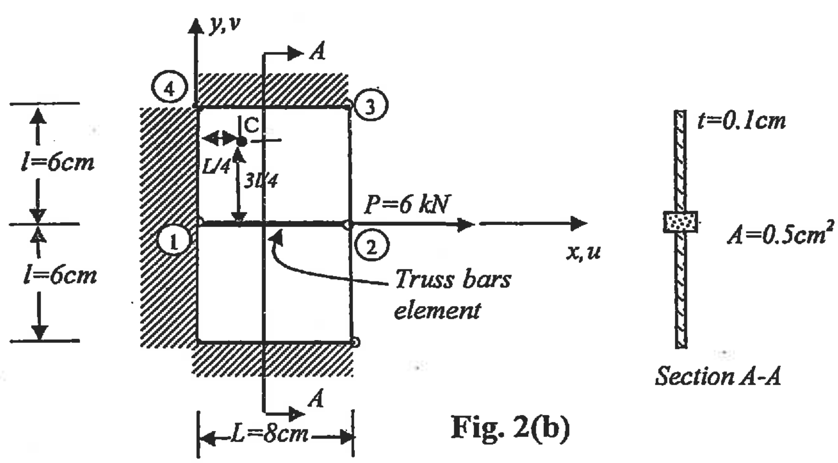


Fig. 2(b)

1) Calculate the horizontal displacement u_4
2) Calculate the strains ε_x, ε_y and γ_{xy} at point C as shown in the Fig. 2(b).

Problem 3

3.1 A square frame subjected to a pair of parting forces, ***P***, as shown in Fig 3(a). It is assumed that the frame members are inextensible and that the right angles at the joints are preserved. Draw the deflections shape, shearing forces diagram and bending moment diagram. The stiffness of each member is denoted ***EI***.

3.2 The frame is now reinforced with four truss bars as shown in Fig. 3(b), Draw the deflections shape, shearing forces diagram and bending moment diagram. The stiffness of each of the truss members is given by: $EA = \frac{12EI\sqrt{2}}{L^2}$.

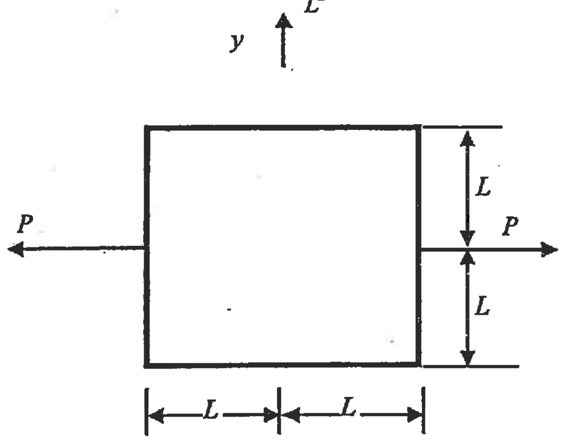


Figure 3(a)

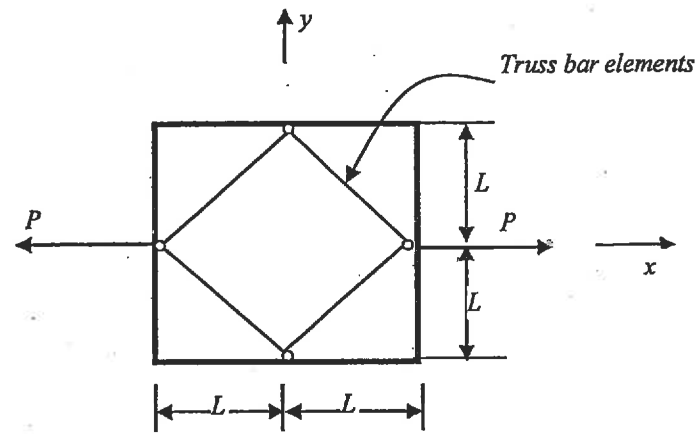


Figure 3(b)

The stiffness matrix of a beam element is given as following:

$$[k] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{EA}{L} & 0 & 0 \\ & \text{SYM} & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix}$$