National Exams December 2013

07-Elec-A2, Systems & Control

3 hours duration

NOTES:

1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a single-sided, handwritten, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet has to be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining five. Clearly indicate answers to which questions should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. You may use exam booklets to answer the questions - please clearly indicate which question is being answered.

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING FIVE QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
TOTAL:	/100	

A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{s^2}$	$t \cdot 1(t)$
$\frac{1}{s^k}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$t \cdot e^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin(at) \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos(at) \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos(bt) \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin(bt) \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1-e^{-at} \cdot \left(\cos bt + \frac{a}{b}\sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cdot \sin(\omega_n \sqrt{1-\xi^2}\, t) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$	$\left(1-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cdot \sin(\omega_n \sqrt{1-\xi^2}\, t + \cos^{-1}\xi)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

Question 1 (Compulsory)

Stability Analysis: Frequency Domain (Gain and Phase Margins) vs. s-Domain (Routh-Hurwitz Criterion).

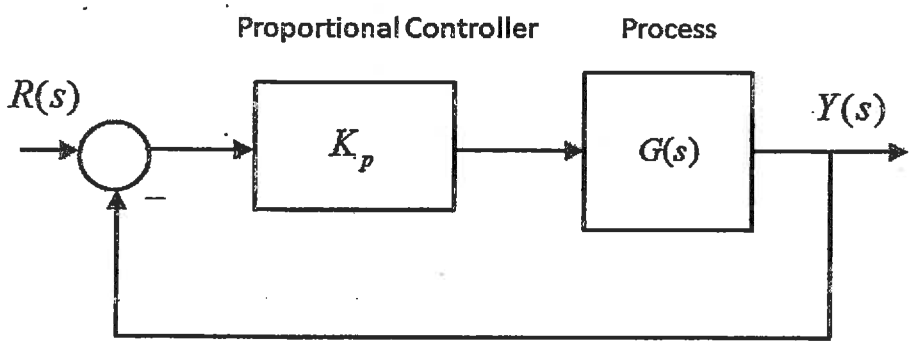

Figure Q1.1

Consider a unit feedback control system shown in Figure Q1.1. The process G(s) is described by the following transfer function:

$$G(s) = \frac{150(s+50)}{(s+3)^2(s+8)}$$

Your task is to investigate the stability of the closed loop system using two approaches (Bode Plots and Routh-Hurwitz Criterion) by finding:

1. The value of the Proportional Controller Gain, $K_p = K_{crit}$, at which the closed loop system becomes marginally stable, and the corresponding frequency of marginally stable oscillations, ω_{osc}.
2. The practical range of safe operating gains for the Proportional Controller in Figure Q1.1;
3. The Gain Margin of the system, both in V/V units and in decibel units, when the Operating Gain of the Controller K_p is set to 0.05 .

Part A (10 marks)

Figure Q1.2 shows an open-loop frequency response of the process, $K_p G(s)$, when the Controller Gain K_p is set to 1. Use this plot to answer the three questions above.

Part B (10 marks)

Apply the Routh-Hurwitz Criterion of Stability to verify the results of Part A. Are the results of both methods consistent?

Open Loop Bode Plots for Question 1 Part A

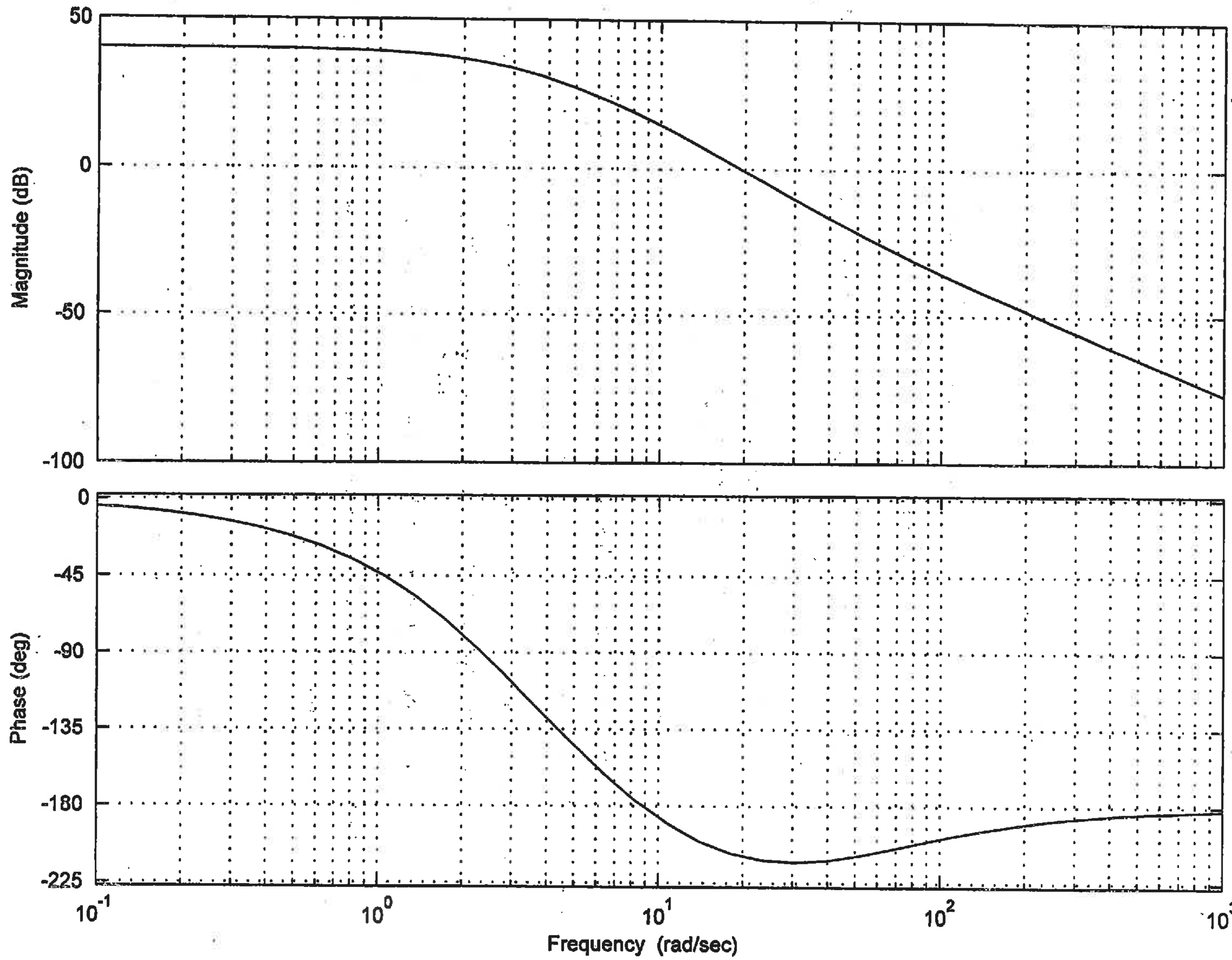

Figure Q1.2

Question 2 (Compulsory)

Root Locus Analysis and Stability Analysis in s-domain.

Consider again a unit feedback control system under Proportional Control shown in Figure Q1.1, where the process G(s) is described by a different transfer function:

$$G(s) = \frac{1}{(s-5)(s^2+6s+40)}$$

Note that the process G(s) is unstable and that it also has a pair of complex poles. Your task is to investigate the behaviour of the closed loop system, including its stability, using Root Locus Analysis.

Part A (10 marks)

Use the space provided in Figure Q2.1 to sketch a detailed Root Locus for the system, including break-away/break-in coordinates, if any, asymptotes, if any, a centroid, angles of departure from the complex poles, etc. If you are using estimates, explain why.

Part B (10 marks)

1. Determine the exact coordinates of the crossovers of the Root Locus with the Imaginary Axis;
2. Next, determine the value(s) of the Proportional Controller Gain, $K_p = K_{crit}$, at which the closed loop system becomes marginally stable, and the corresponding frequency(ies) of marginally stable oscillations, ω_{osc}.
3. Finally, determine the practical range of safe operating gains for the Proportional Controller in this system.

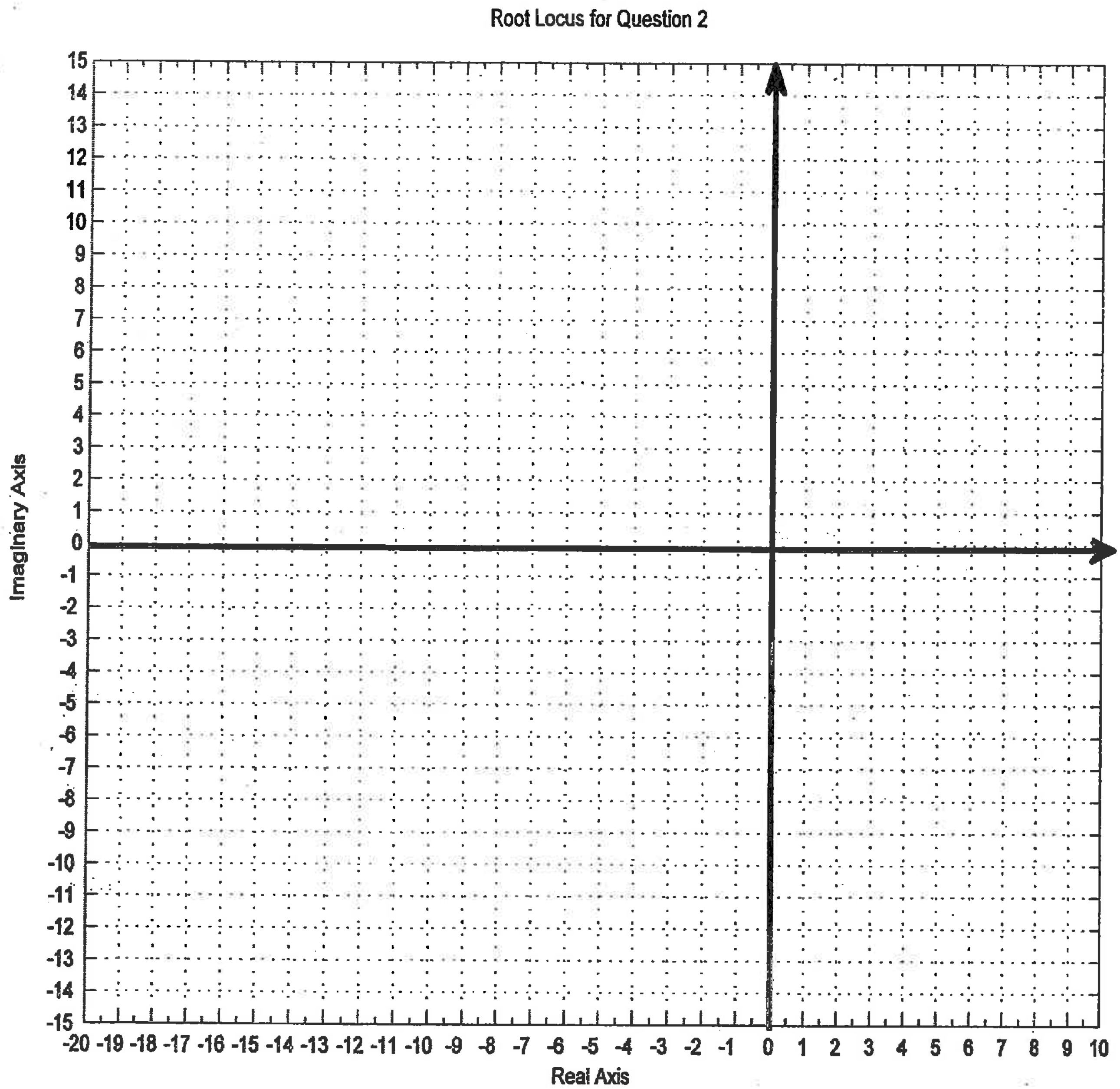
Root Locus for Question 2
Imaginary Axis
Real Axis
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
-13
-14
-15
-20 -19 -18 -17 -16 -15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Figure Q2.1

Question 3

Proportional + Integral (PI) Controller Design in s-Domain, Dominant Poles Model, Step Response Specifications.

Consider a unit feedback closed loop control system, as shown in Figure Q3.1.

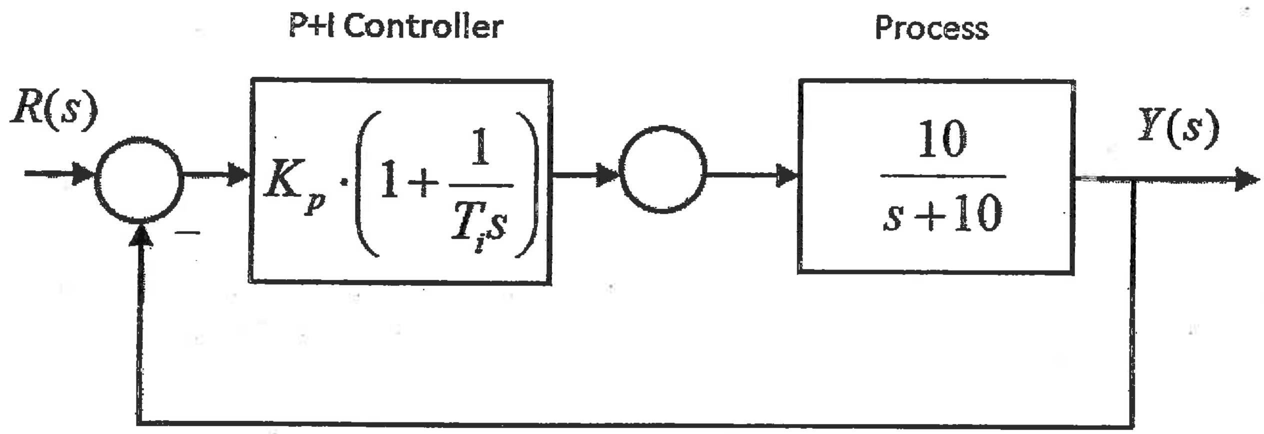

Figure Q3.1

The system is to operate under PI Control. Your task is to calculate the PI Controller parameters, K_p, and T_i. In order to do so, please follow the steps described in the following parts.

PART A (7 marks)

Assume that the response can be modeled by a standard second order dominant poles model that is described by the following transfer function:

$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Determine appropriate parameters of the model (i.e. K_{dc}, ω_n, ζ) so that the compensated closed loop response of our control system would have the following specifications:

- Percent Overshoot, $PO = 15\%$
- Settling time within 2% of the steady state value, $T_{settle(\pm 2\%)} = 0.1$ seconds
- Steady State Error, $e_{ss\%} = 0$

PART B (7 marks)

Derive the closed loop transfer function of the PI-compensated system, $G_{cl}(s) = \frac{Y(s)}{R(s)}$, in terms of K_p and T_i. Next, calculate appropriate numerical values for the PI Controller parameters K_p and T_i, so that the specifications from Part A are met.

PART C (6 marks)

Substitute the computed values of the controller parameters, K_p and T_i into the closed loop transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$.

Compare the two closed loop transfer functions, $G_{cl}(s)$ and $G_m(s)$. What is the difference between them, and how will it affect each of the three specifications from Part A? Please describe briefly.

Question 4

Proportional + Derivative (PD) and Proportional + Rate Feedback Controller Design in s-Domain, Dominant Poles Model, Step Response Specifications.

Consider a closed loop control system operating under a Proportional + Rate Feedback Control, as shown in Figure Q4.1.

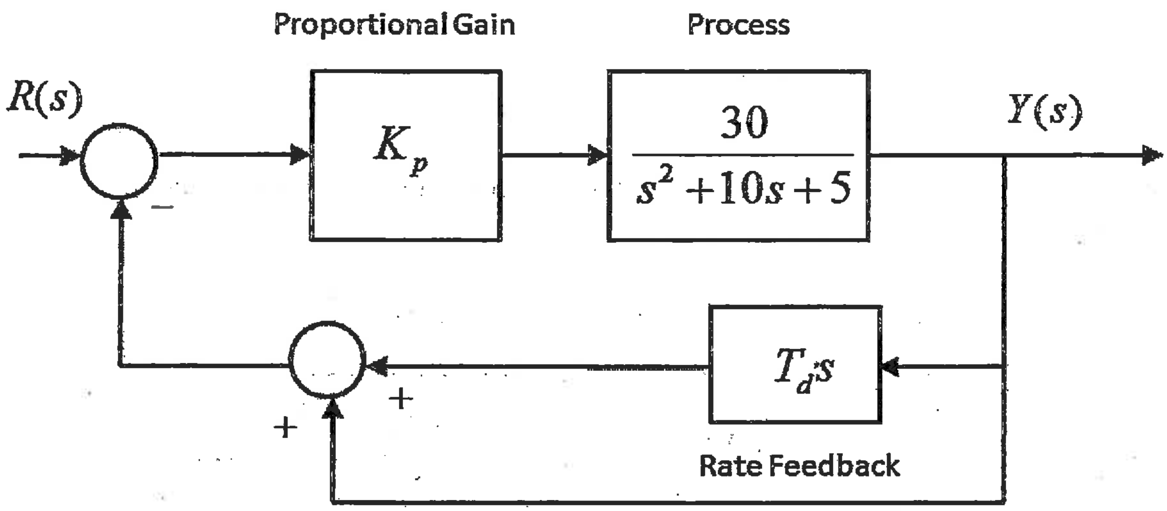

Figure Q4.1

PART A (8 marks)

Find the closed loop transfer function of the compensated system, in terms of the controller parameters K_p and T_d. Are there any restrictions on the values of these two parameters with respect to the closed loop system stability? If so, please specify them. Next, calculate the values for K_p and T_d so that:

- The closed loop system step response error, in %, is: $e_{ss\%} = 5\%$
- The closed loop damping ratio is: $\zeta = 0.7$

Substitute the controller parameter values K_p and T_d and calculate the closed loop transfer function of the compensated system, $G_{cl1}(s)$.

PART B (4 marks)

Estimate the following closed loop step response specifications:

- Percent Overshoot, PO
- Settling time within 2% of the steady state value, $T_{settle(\pm 2\%)}$
- Steady State Error, $e_{ss\%}$

PART C (8 marks)

Next, consider the modified closed loop system operating under a Proportional + Derivative Control, as shown in Figure 4.2.

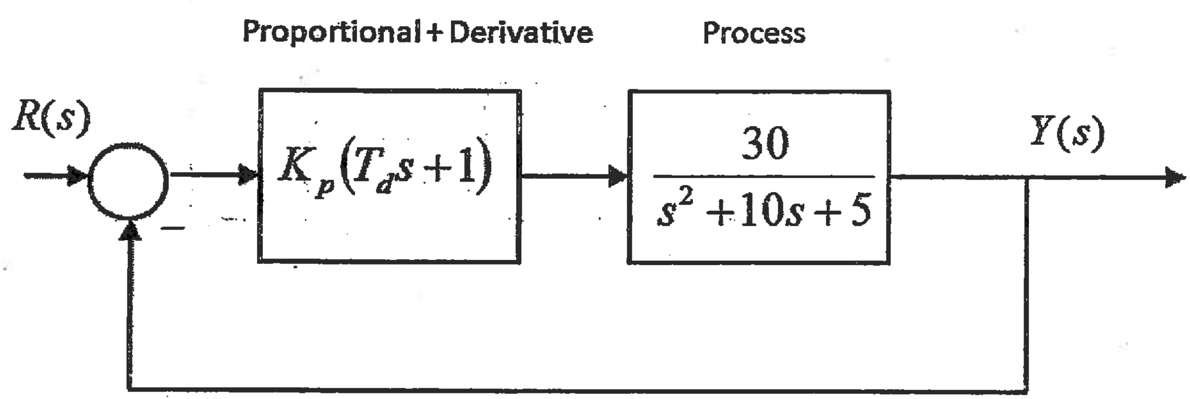

Figure Q4.2

Assume the same values of the controller parameters K_p and T_d as in Part A, and calculate the closed loop transfer function of the modified compensated system, $G_{cl2}(s)$. Compare the two closed loop transfer functions, $G_{cl1}(s)$ and $G_{cl2}(s)$.

What is the difference between them, and how will it affect each of the three specifications from Part B? Please describe briefly.

Question 5

Proportional vs. Lead Control, Closed Loop Stability: Gain and Phase Margins, Second Order Dominant Poles Model from Frequency Response Plots, Step Response Specifications, Steady State Errors and Error Constants.

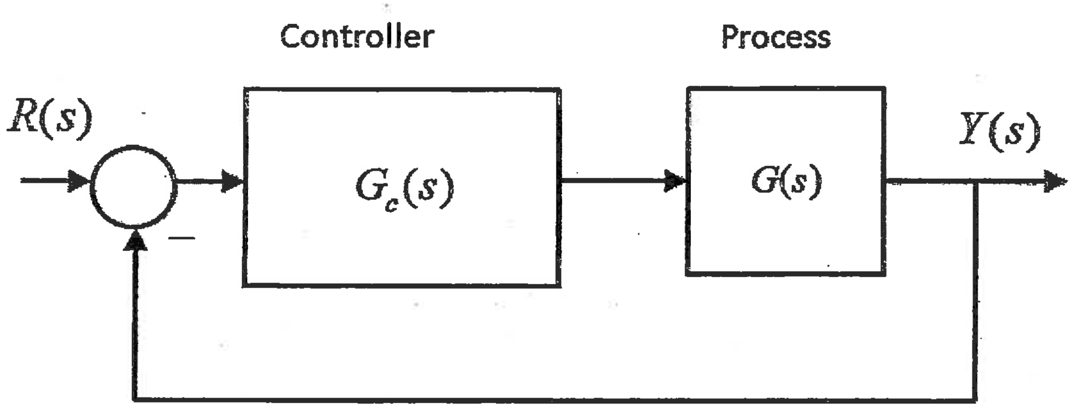

Figure Q5.1

Consider a certain unit feedback closed loop system, shown in Figure Q5.1. The process G(s) is described by the following transfer function:

$$G(s) = \frac{1}{s(s+1)(s+2.7)}$$

Open loop frequency response plots for the process G(s) only (i.e. without controller) are shown in Figure Q5.2. The Gain and Phase Margins and their corresponding crossover frequencies are as indicated.

PART A (5 marks)

Assume first that the closed loop system would operate under Proportional Control. Find the required Proportional Controller gain value, K_p, such that closed loop system error to a unit ramp input is equal to 0.27 V/V. Evaluate the closed loop system stability. Is the Proportional Controller adequate?

PART B (10 marks)

Next, assume that the closed loop system would operate under a Lead Controller described by the following transfer function:

$$G_c(s) = K_c \frac{\tau s + 1}{\alpha \tau s + 1} \qquad \alpha < 1$$

Design the appropriate Lead Controller for this system, so that :

- The compensated closed loop system response meets the closed loop system error to a unit ramp input from Part A
- The closed loop system is stable and has the following Phase Margin: $\Phi_m = +45^0$
- The compensated crossover frequency for the Phase Margin is: $w_{cp} = 3$ rad/sec.

Make sure to show the approximate shape of the compensated open loop frequency response in Figure Q5.2.

PART C (5 marks)

Assume that the compensated closed loop system can be modeled by a standard second order dominant poles model that is described by the following transfer function:

$$G_m(s) = K_{dc} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Determine appropriate parameters of the model (i.e. K_{dc}, ω_n, ζ) and estimate the following closed loop step response specifications: Steady State Error, $e_{ss\%}$, Percent Overshoot, PO, and Settling Time, $T_{settle(\pm 2\%)}$.

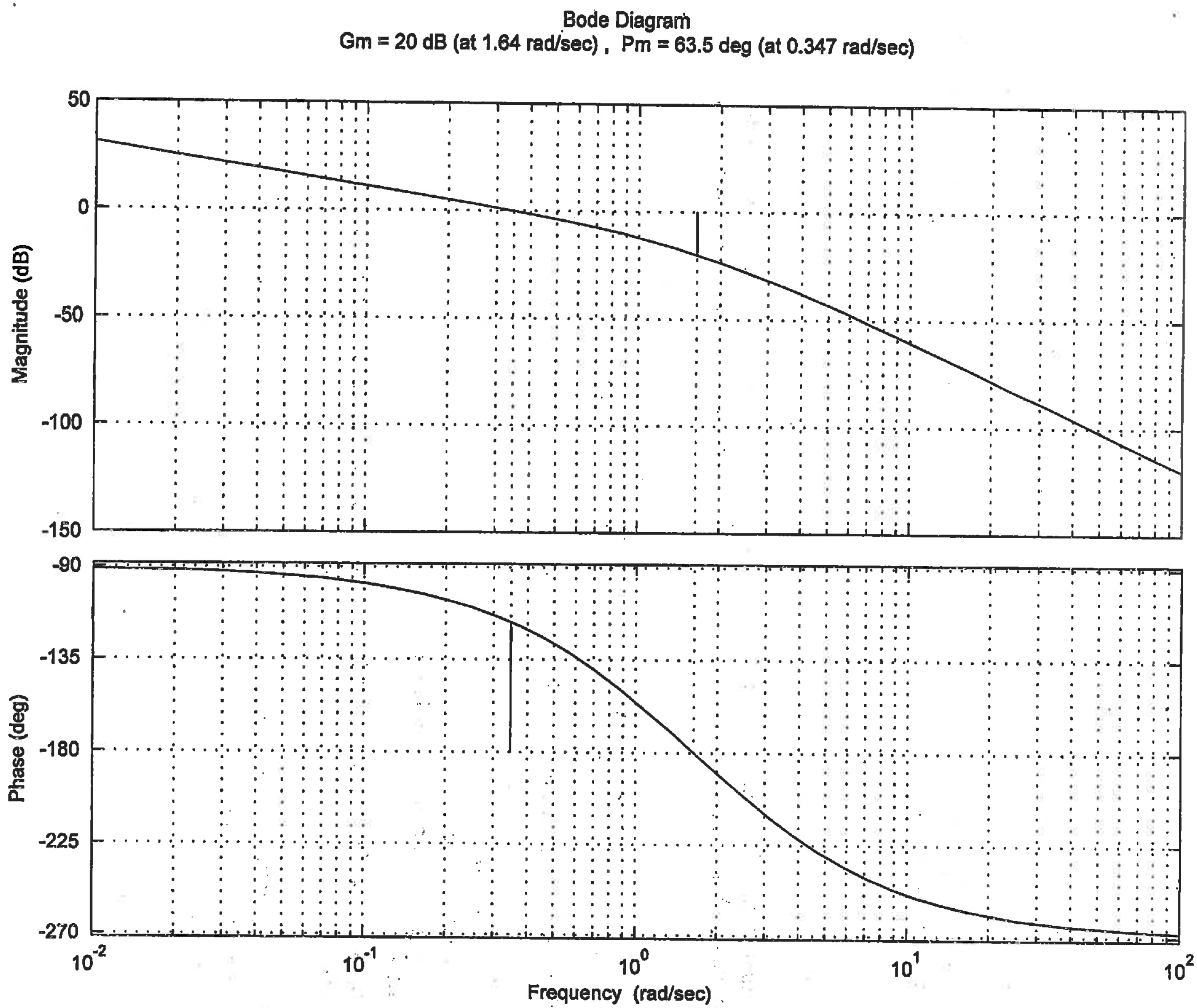
Bode Diagram
Gm = 20 dB (at 1.64 rad/sec) , Pm = 63.5 deg (at 0.347 rad/sec)
Magnitude (dB)
50
0
-50
-100
-150
Phase (deg)
-90
-135
-180
-225
-270
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)

Figure Q5.2

Question 6

Step Response Specifications, Standard Second Order Model, Effect of Additional Poles and Zeros on the Step Response.

PART A (12 marks)

A certain *4^th^* order system has one zero at -12 and its poles are located at the following coordinates in the s-plane: -20, -16, -1-j5, -1+j5. The system DC gain is equal to 0.95.

a) **(4 marks)** Write a polynomial expression for the system transfer function, G(s), in a standard form as shown below:

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

b) **(4 marks)** Briefly explain why a *2^nd^* order transfer function is an appropriate choice for the system model. Write the model transfer function $G_m(s)$ in a standard form as shown below:

$$G_m(s) = K_{dc} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Clearly indicate values of the model parameters:

- DC gain, K_{dc}
- Damping ratio, ζ
- Frequency of natural oscillations, ω_n.

c) **(4 marks)** Use the *2^nd^* order model $G_m(s)$ to estimate the following system step response specifications:

- Steady State Error in %, $e_{ss\%}$
- Percent Overshoot, PO
- Settling Time (within 2% of the steady state), $T_{settle(\pm 2\%)}$

PART B (8 marks)

Consider a step response of a certain process $G(s)$ to a unit reference signal, shown in Figure Q6.1 on the next page. Assume a second order dominant poles model for the process G(s) and find the model parameters:

- DC gain, K_{dc}
- Damping ratio, ζ
- Frequency of natural oscillations, ω_n.

Finally, write the model transfer function $G_m(s)$:

$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

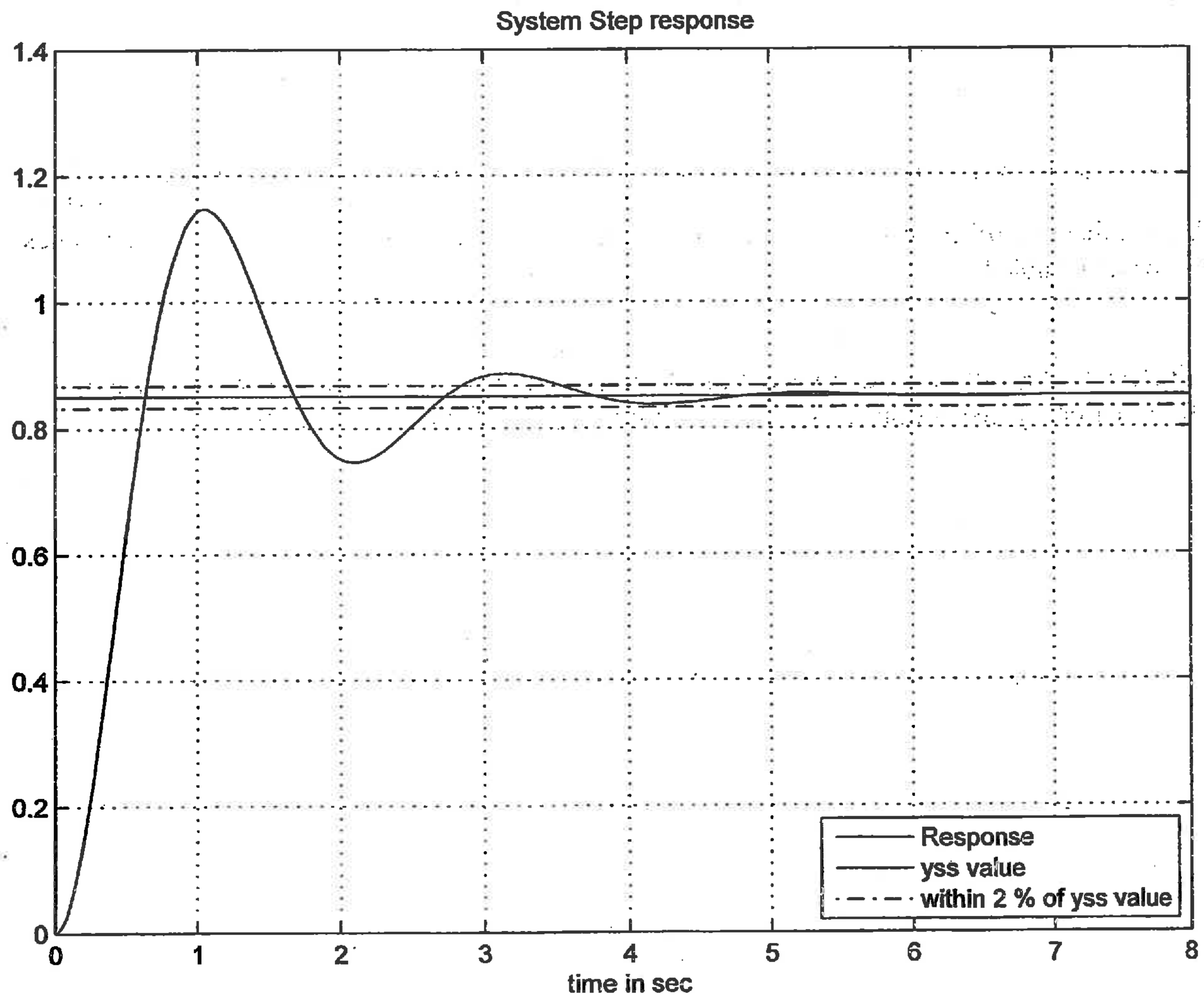

Figure Q6.1

Question 7

Transfer Function vs. State Space Model, Canonical Forms, System Eigenvalues, Stability.

Part A (10 marks)

Consider a control system described by the following state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -9 & 1 & 0 \\ -23 & 0 & 1 \\ -15 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$

The system equations are in a Canonical Form. Find the system transfer function $G(s) = \dfrac{Y(s)}{U(s)}$.
It is known that one of the system eigenvalues is equal to -5. Find the remaining two eigenvalues - is the system stable?

HINT: Because the system equations are in a Canonical Form, you should be able to write the transfer function by inspection, without calculating the Transfer Function Matrix.

Part B (10 marks)

A certain Single Input Single Output (SISO) control system is described by the following transfer function:

$$G(s) = \frac{3s^2 + 2s + 1}{s^3 + 7s^2 + 2s + 15}$$

a) **(5 marks)** Is the system stable? Justify your answer.

b) **(5 marks)** Find a Controllable Canonical Form of the state space realization of the system transfer function. Clearly identify matrices **A**, **B**, **C**, and **D** of the general state space model as written in a standard form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$