National Exams December 2013

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.

3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Question 1:

a) Calculate the unit step response of the system

$$G(s) = \frac{54}{(2s+6)(s^2+3s+9)}$$

b) A system is given by its transfer function

$$G(s) = \frac{5(1-0.4s)}{(s+1)(0.2s+1)}$$

What are the time constants of the components of its transient response, and how long does it take for the transient to decay almost completely?

Question 2:

For the system of Fig. 1 with $G(s) = 1/[s(s+1)(s+4)]$, use the Routh-Hurwitz criterion to find the limit on K for stability. At this limit, determine the position of the system poles on the imaginary axis of the s-plane.

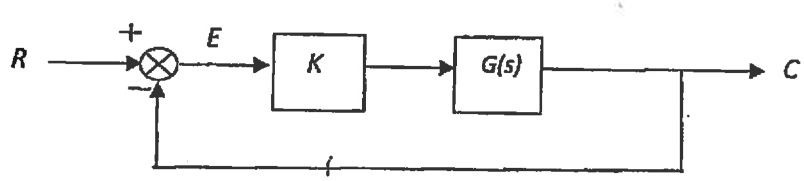

Figure 1

Question 3:

For the system in Figure 2 with $G(s) = (s + 1)(s + 3)/ [s(s + 2)(s + 4)]$:

(a) What is the system type number?

(b) What is the gain of the loop gain function?

(c) What are the steady-state errors following unit step and unit ramp inputs?

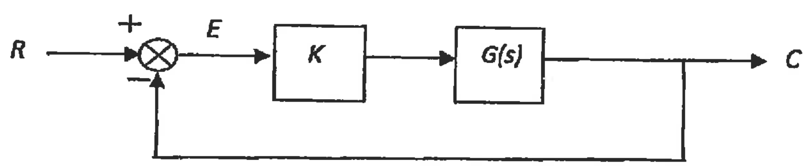

Figure 2

Question 4:

(a) For the motor position servo in Fig. 3 without the rate or velocity feedback ($K_g = 0$), find K for a system damping ratio 0.5 and the corresponding steady-state error following unit ramp inputs.

(b) Still with $K_g = 0$, what value of K will give a steady-state unit ramp following error of 0.1, and what is corresponding damping ratio?

(c) With K as in part (b), what value of K_g will give a system damping ratio 0.5? How does the steady-state error compare with that in part (b)?

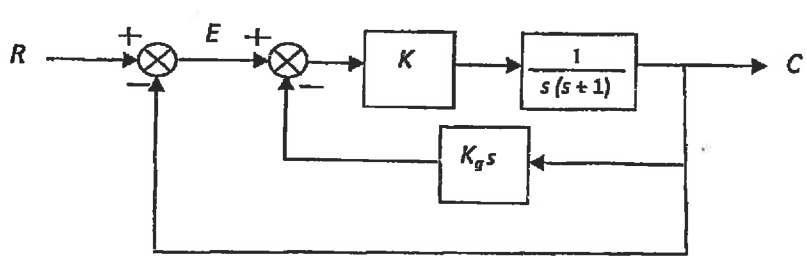

Figure 3

Question 5:

For a system with loop gain function

$$G = \frac{K\,(s^2 + 4s + 8)}{s^2\,(s - 1)}$$

sketch the loci and find the range of values of K for which the system is stable.

Question 6:

For a system with loop gain function

$$G(s) = \frac{20}{(s + 5)\,(0.1s + 1)\,(0.025s + 1)}$$

(a) Plot the asymptotic Bode magnitude plot.

(b) Determine the phase margin and gain margin.

Laplace Transform Table

Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n (n = positive integer)
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})$ $(\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(\beta e^{-\beta t}-\alpha e^{-\alpha t})$ $(\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha te^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t-1+e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t-\frac{1}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2+\omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2+\omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos \omega_n t$
$\frac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n\sqrt{\alpha^2+\omega_n^2}\sin(\omega_n t+\theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s+\alpha)(s^2+\omega_n^2)}$	$\frac{\omega_n}{\alpha^2+\omega_n^2}e^{-\alpha t}+\frac{1}{\sqrt{\alpha^2+\omega_n^2}}\sin(\omega_n t-\theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}\,t$ $(\zeta<1)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta = \cos^{-1}\zeta$ $(\zeta<1)$
$\frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\theta)$ where $\theta = \cos^{-1}\zeta$ $(\zeta<1)$
$\frac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\frac{\alpha^2-2\alpha\zeta\omega_n+\omega_n^2}{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta = \tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\alpha-\zeta\omega_n}$ $(\zeta<1)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n^2\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta = \cos^{-1}(2\zeta^2-1)$ $(\zeta<1)$

12-184

Semi-Logarithmic
4 Cycles x 10 to the inch

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