# National Exams December 2013

## Communications

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a Closed Book Exam but one aid sheet is allowed written on both sides. An approved calculator is permitted.
3. There are six questions, however, FIVE (5) questions constitute a complete paper. The first five questions as they appear in the answer book will be marked.
4. All questions are of equal value.
5. Clarity and organization of the answer are important.

1. **(Total 20 marks)** A periodic signal $x(t)$ is expressed as

$$x(t) = 2\cos t + \cos(3t - \frac{2\pi}{3}) + 2\cos(8t + \frac{2\pi}{3})$$

(a) **(10 marks)** Sketch the amplitude and phase spectra for the trigonometric Fourier series.

(b) **(5 marks)** By inspection of spectra in part (a), sketch the exponential Fourier series spectra.

(c) **(5 marks)** By inspection of spectra in part (b), write the exponential Fourier series for $x(t)$.

2. **(Total 20 marks)** Over an interval $|t| \leq 1$, an angle modulated signal is given by

$$\varphi_{EM}(t) = 10 \cos 12,000t$$

It is known that the carrier frequency $\omega_c = 10,000$. The $m(t)$ is modulating signal.

(a) **(10 marks)** If this were a PM (phase modulation) signal with $k_p = 1000$, determine $m(t)$ over the interval $|t| \leq 1$.

(b) **(10 marks)** If this were an FM (frequency modulation) signal with $k_f = 1000$, determine $m(t)$ over the interval $|t| \leq 1$.

3. **(20 marks total)** A signal $x(t) = sinc^2(10\pi t)$ is sampled (using uniformly spaced impulses) at a rate of (i) 10 Hz; (ii) 20 Hz; (iii) 30 Hz. For each of the three case:

   (a) **(5 marks)** Sketch the sampled signal.

   (b) **(5 marks)** Sketch the spectrum of the sampled signal.

   (c) **(5 marks)** Explain whether you can recover the signal $x(t)$ from the sampled signal.

   (d) **(5 marks)** If the sampled signal is passed through an ideal low-pass filter of bandwidth 10 Hz, sketch the spectrum of the output signal.

4. **(Total 20 marks)** Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ depicted in the Figure 1.

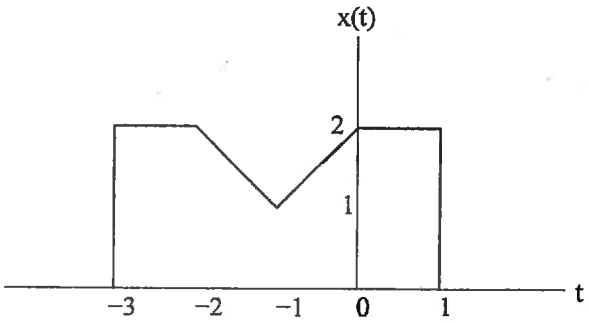


Figure 1:

(a) **(4 marks)** Find $\angle X(j\omega)$.

(b) **(4 marks)** Find $X(j0)$.

(c) **(4 marks)** Find $\int_{-\infty}^{\infty} X(j\omega)d\omega$.

(d) **(4 marks)** Evaluate $\int_{-\infty}^{\infty} X(j\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}d\omega$.

(e) **(4 marks)** Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$.

(You may use properties of the Fourier transform to perform all these calculations)

5. (**Total 20 marks**) Consider a right-sided sequence $x[n]$ with z-transform

$$X(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - z^{-1})}.$$

(a) (**10 marks**) Carry out a partial-fraction expansion of the equation given above which is expressed as a ratio of polynomials in $z^{-1}$, and from this expansion determine $x[n]$.

(b) (**10 marks**) Rewrite the equation $X(z)$ given above as a ratio of polynomials in $z$, and carry out a partial-expansion of $X(z)$ expressed in terms of polynomials in $z$. From this expansion, determine $x[n]$, and demonstrate that the sequence obtained is identical to that obtained in part (a).

6. **(Total 20 marks)** Consider a radio transmitter rated for $S_T \leq 3$ kW ( $S_T$ is average transmitted power) and $A_{max}^2 \leq 8$ kW ($A_{max}^2$ is peak envelope power). Let the modulating signal be a tone with $A_m = 1$

   (a) **(5 marks)** What is $S_x$, message power?

   (b) **(5 marks)** If the modulation is DSB, what is the maximum possible power per sideband ($P_{sb}$)?

   (c) **(10 marks)** Let the modulation signal be a square wave that switches periodically between $x(t) = +1$ and $x(t) = -1$. Sketch $x_c(t)$ when

      i. the modulation is AM with $\mu = 0.5$ ($\mu$ is modulation index),

      ii. the modulation is AM with $\mu = 1$, and

      iii. the modulation is DSB.

   Indicate the envelopes by dashed lines.