

National Exams May 2013  
04-BS-1, Mathematics  
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
  2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
  3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
  4. All questions are of equal value.
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Marking Scheme:

1. 20 marks
2. (a) 7 marks, (b) 7 marks, (c) 6 marks
3. (a) 6 marks, (b) 14 marks
4. 20 marks
5. 20 marks
6. (a) 7 marks, (b) 7 marks, (c) 6 marks
7. 20 marks
8. 20 marks

1. Solve the initial value problem

$$y'' - 4y' + 8y = 5 \cos(2t), \quad y(0) = 0, \quad y'(0) = 0,$$

where ' denotes differentiation with respect to  $t$ .

2. Find the general solution of the following differential equations:

(a)  $y' + 2xy = 2xe^{-x^2}$ ,

(b)  $y' + 2xy^2 = 0$ ,

(c)  $y'' - 2y' + 3y = 0$ .

3. (a) Find the eigenvalues and the eigenvectors of the matrix  $A = \begin{pmatrix} 4 & 3 \\ -1 & 0 \end{pmatrix}$ .

- (b) Solve the system of differential equations

$$\frac{dx}{dt} = 4x + 3y,$$

$$\frac{dy}{dt} = -x + t,$$

subject to the initial conditions  $x(0) = 2, y(0) = -1$ .

4. Let  $f(x, y) = 1 + x \ln(xy - 5)$ . Find a formula for the plane tangent to the surface  $z = f(x, y)$  at the point  $(2, 3)$  and use the tangent plane to approximate  $f(2.1, 2.95)$ .

5. Find the surface area of that portion of the surface  $z = 1 - \sqrt{x^2 + y^2}$  that lies in the first octant.

6. Consider the two lines defined as follows:

$$\begin{aligned} x &= 3 - 2t, & y &= 3, & z &= 3 - t, & (\text{parameter } t); \\ x &= s, & y &= 1 - 2s, & z &= -s, & (\text{parameter } s). \end{aligned}$$

- (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.

- (b) Find a third line orthogonal to both lines.

- (c) Is there a plane containing both lines? If so, find an equation for that plane.

7. Evaluate the line integral  $\oint_C \mathbf{v} \cdot d\mathbf{r}$  where  $C$  is the curve formed by the intersection of the cylinder  $x^2 + y^2 = 4$  and the plane  $z + 2x - y = 3$ , travelled counterclockwise as viewed from the positive  $z$ -axis, and  $\mathbf{v}$  is the vector function  $\mathbf{v} = x\mathbf{i} + (x - y)\mathbf{j} + yz\mathbf{k}$ .

8. Find the volume of the solid region inside the ellipsoid

$$x^2 + y^2 + 4z^2 = 5$$

and above the cone

$$z = \sqrt{x^2 + y^2}.$$