

National Exams December 2014
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
 2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
 3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
 4. All questions are of equal value.
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Marking Scheme:

1. (a) 10 marks, (b) 10 marks
2. 20 marks
3. (a) 7 marks, (b) 7 marks, (c) 6 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. (a) 6 marks, (b) 6 marks, (c) 8 marks
8. 20 marks

1. For each of the following differential equations, find the general solution, $y(x)$.

(a) $y'' + 4y = \sec 2x$

(b) $y'' + y' - 6y = 3x^2 + e^{2x}$

Note that ' denotes differentiation with respect to x .

2. Find the general solution, $y(x)$, of the differential equation

$$2x^2y'' + xy' - 3y = \frac{4}{x}.$$

Note that ' denotes differentiation with respect to x .

3. Consider the two lines defined as follows:

$$x = 2 - t, \quad y = 3t, \quad z = 1 + t, \quad (\text{parameter } t);$$

$$x = 1 + s, \quad y = 3 - 2s, \quad z = 2 + 4s, \quad (\text{parameter } s).$$

- (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.
 (b) Find a third line orthogonal to both lines.
 (c) Is there a plane containing both lines? If so, find an equation for that plane.

4. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = 4x\mathbf{i} + 2x^2\mathbf{j} - 3\mathbf{k},$$

S is the surface of the region bounded by the cone $z = 4 - \sqrt{x^2 + y^2}$ and the plane $z = 0$.

5. Let C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = 1 + y$, and let \mathbf{v} be the vector function $\mathbf{v} = 4z\mathbf{i} - 2x\mathbf{j} + 2x\mathbf{k}$. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$. Assume a clockwise orientation for the curve when viewed from above.
6. Find the volume of the solid region outside the cylinder $x^2 + y^2 = 1$, but inside the ellipsoid $x^2 + y^2 + 4z^2 = 4$.
7. Consider the matrix

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 1 & 0 \\ -10 & -4 & -2 \end{pmatrix}$$

- (a) Show that $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ is an eigenvector of A and find the associated eigenvalue.

(b) Show that 3 is an eigenvalue of A and find an associated eigenvector.

(c) Solve the linear system $\mathbf{x}' = A\mathbf{x}$ for the function $\mathbf{x}(t)$.

8. Let $f(x, y) = 1 + x \ln(xy - 5)$. Find a formula for the plane tangent to the surface $z = f(x, y)$ at the point $(2, 3)$ and use the tangent plane to approximate $f(2.1, 2.95)$.