NATIONAL EXAMINATIONS

December 2014

04-BS-7 MECHANICS OF FLUIDS

Three (3) hours duration


Notes to Candidates

1.   This is a **Closed Book** examination.

2.   Exam consists of two Sections. **Section A is Calculative (9 questions)** and **Section B is Analytical (4 questions)**.

3.   **Do seven (7) questions from Section A (Calculative) and three (3) questions from Section B (Analytical).** Note that the Analytical Questions do not require detailed calculations but do require full explanations.

4.   **Ten (10) questions constitute a complete paper.** (Total 50 marks).

5.   **All questions are of equal value.** (Each 5 marks).

6.   If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.

7.   Candidates may use one of the approved **Casio** or **Sharp** calculators.

8.   **Reference information** for particular questions are given on pages 7 to 10. **All pages of questions attempted are to be returned with the Answer Booklet showing diagrams generated or where readings were taken and which data was used. Candidates must write their names on these pages.**

9.   **Constants** are given on page 11.

10.   **Reference Equations** are given on pages 12 to 15.

SECTION A   CALCULATIVE QUESTIONS

**Do seven of nine questions.**   Solutions to these questions must be set out logically with all intermediate answers and units given.

QUESTION 1

Refer to the adjacent figure. The hemispherical body shown in the figure projects into a tank. Find the horizontal and vertical forces and hence the total force acting on the hemispherical projection for the case when the tank is full of water with the free surface 2.5 m above the centre of the hemisphere (Point A). The radius of the hemisphere is 1 m.
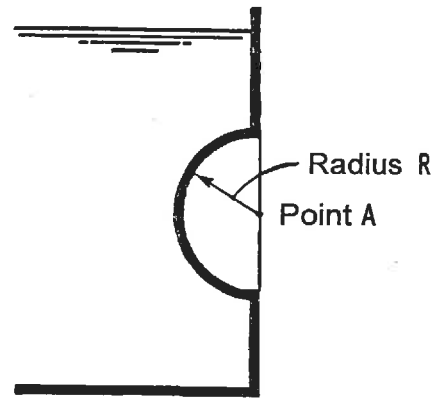
Hint: Consider buoyant effect of hemisphere

(5 marks)

QUESTION 2

A gravity dam built of concrete has dimensions as shown in the adjoining sketch. It is filled with water to a depth of 18 m. To prevent cracking of the concrete or separation from the bedrock, the resultant force on the structure must pass through the middle third of the base so as to maintain compressive stress everywhere in the concrete and bedrock. By taking moments about the toe of the dam (Point O), determine the point (horizontal distance from Point O) through which the resultant force passes through the base and hence whether the dam is safe or not.
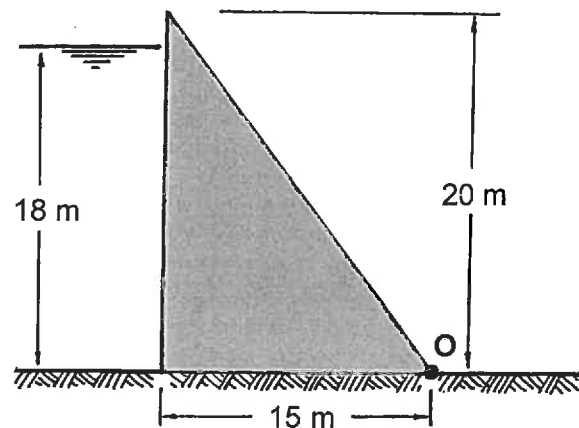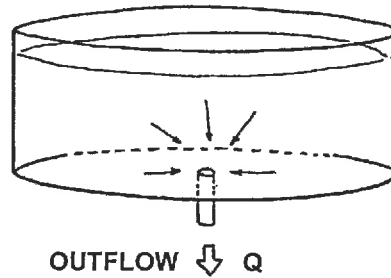
(5 marks)

QUESTION 3

A submersible package fountain (motor, pump and jet in a single unit) is required for a garden pond. The jet is to be 20 mm in diameter and is to shoot the water to a height of 2 m. When submersed the nozzle is just above the water surface. Assuming that the pump and hydraulic components have an overall efficiency of 65%, determine the power required to run the fountain.

(5 marks)

QUESTION 4

Water flows at a rate of 0.300 L/s through a small circular hole in the bottom of a large tank. Assuming the water in the tank approaches the hole <u>radially</u>, calculate the velocity in the tank at a distance of 100 mm from the hole.
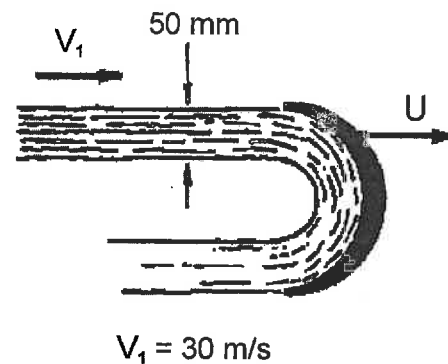
Hint: Consider flow through an imaginary hemisphere inside the tank.

OUTFLOW ⇩ Q

(5 marks)

QUESTION 5

A jet of water with a diameter of 50 mm and a velocity V_1 of 30 m/s strikes a curved plate and is turned completely through 180° without friction loss. The plate is driven by the jet and in the same direction as the jet.

(a) Determine the plate speed U for maximum efficiency of energy transfer.
(b) Calculate the force exerted on the plate.
(c) Calculate the power developed by the plate.
(d) Determine the efficiency of energy transfer.

V_1

50 mm

U

$V_1 = 30$ m/s

(5 marks)

QUESTION 6

A motor cyclist without a face mask has his mouth facing directly forward and hence breathes in and out at the stagnation point. Determine at what speed the stagnation pressure would be so great as to prevent the rider from exhaling (and possibly suffocating). Assume that his maximum breathing out pressure is equivalent to 100 mm water gauge.

*Hint: Use energy equation.*

V_2
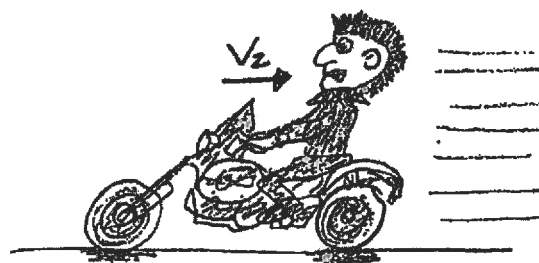
Drawn by Daniel LeBlanc Second Year Student 2003

(5 marks)

QUESTION 7

Refer to the Examination Paper Attachments Page 7 **Moody Diagram**

A sloping underground tunnel provides ventilation to the working face of a coal mine. The tunnel is 2.6 m wide and 3.2 m high and slopes from the surface to the mining area over a distance of 1.5 km. The tunnel is unlined and the surface has an irregular profile with rock projections and depressions of 10 cm on either side of the nominal profile. Air at a temperature of 15°C flows through the tunnel at a mean velocity of 8 m/s. Determine the pressure drop in the tunnel.


10 cm

20 cm

10 cm

3.2 m

2.6 m
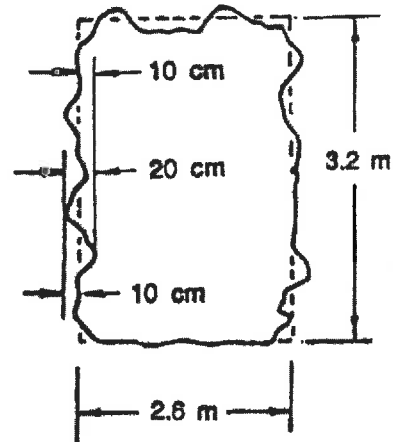
(5 marks)

QUESTION 8

Refer to the Examination Paper Attachments Page 8 **Moody Diagram.**

Consider a concrete trapezoidal irrigation canal with cross section as shown that is 4.5 m wide by 2 m deep with 45° sides and 0.5 m wide bottom. Water is required to flow 1.5 m deep at a flow rate of 3 m³/s. This canal is required to deliver water to a location 8 km from the source. Determine from general pipe flow relations the drop in elevation required to maintain the specified flow rate. Assume a roughness of 1 mm for the concrete.

1.5 m

2 m

0.5 m

2 m
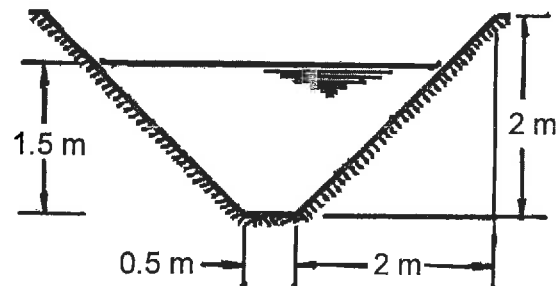
(5 marks)

QUESTION 9

Refer to the Examination Paper Attachments Page 9 **Drag Coefficients for Spheres**

A steel sphere with a specific gravity of 7.8 and a diameter of 6 mm is released in a large tank of oil with a specific gravity of 0.825. The sphere is observed to have a terminal velocity of 0.5 m/s. Calculate the absolute viscosity of the oil.

(5 marks)

SECTION B   GRAPHICAL AND ANALYTICAL QUESTIONS

Do three of four questions. These questions do not require detailed calculations but complete written explanations must be given to support the answers where descriptive answers are required.
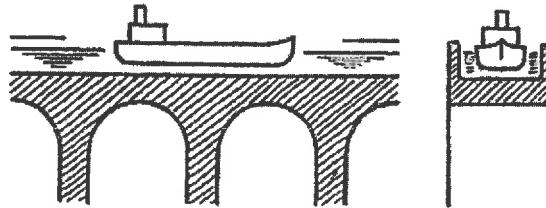
QUESTION 10

Horizontal   Vertical
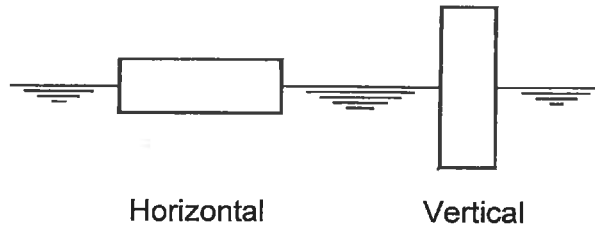
A bar of square cross section with density half that of water will float half submerged as shown in the sketch above. State which orientation will be stable - horizontal or vertical. Explain why the chosen orientation is stable and how this can be proven.

(5 marks)

QUESTION 11

A barge 15 m long and 3 m wide is loaded such that its draught (depth of bottom below water surface) is 1.2 m. It sails in a long canal 5 m wide and 2 m deep. At one point the canal is taken across a valley by an aqueduct as shown in the sketch above. Determine the <u>change</u> in compressive force on the aqueduct pillars as the barge passes over them. Give a full explanation of your answer.

(5 marks)

QUESTION 12

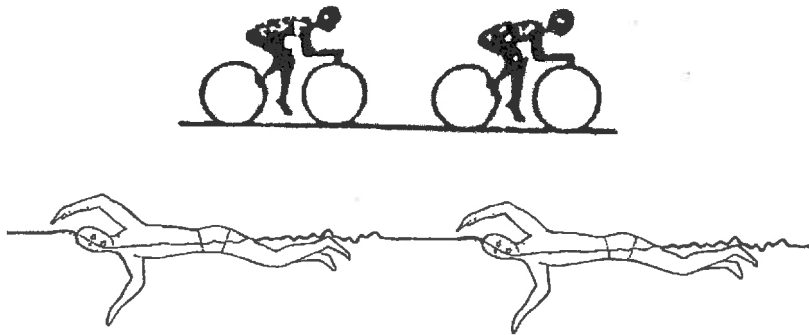Refer to the Examination Paper Attachments Page 10 **Aircraft Wing**

The attached diagram shows an aircraft wing in different configurations. State when and why these different configurations (A, B, C) are used. Explain the physical phenomena contributing to high lift conditions.

(5 marks)

QUESTION 13

Cyclists get some energy advantage by drafting (following closely behind another cyclist). Does the same apply to swimmers who follow closely behind another? By analyzing the forces applicable to cyclists and swimmers determine whether it is advantageous or not for a swimmer to follow closely behind another. Explain your answer fully.

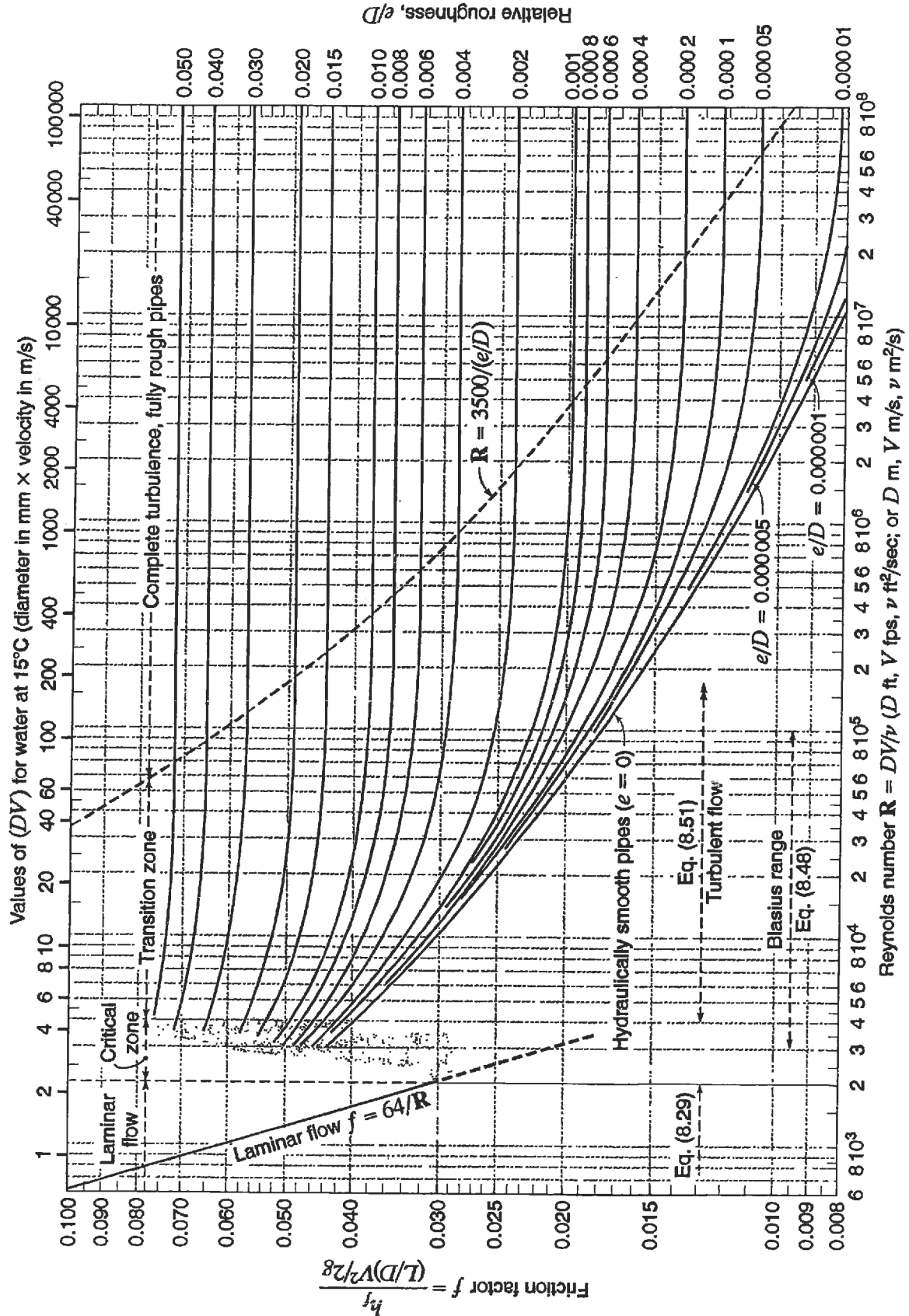*Hint: Use momentum theory and analyse motion and forces related to swimmers.*

(*5 marks*)

EXAMINATION PAPER ATTACHMENTS

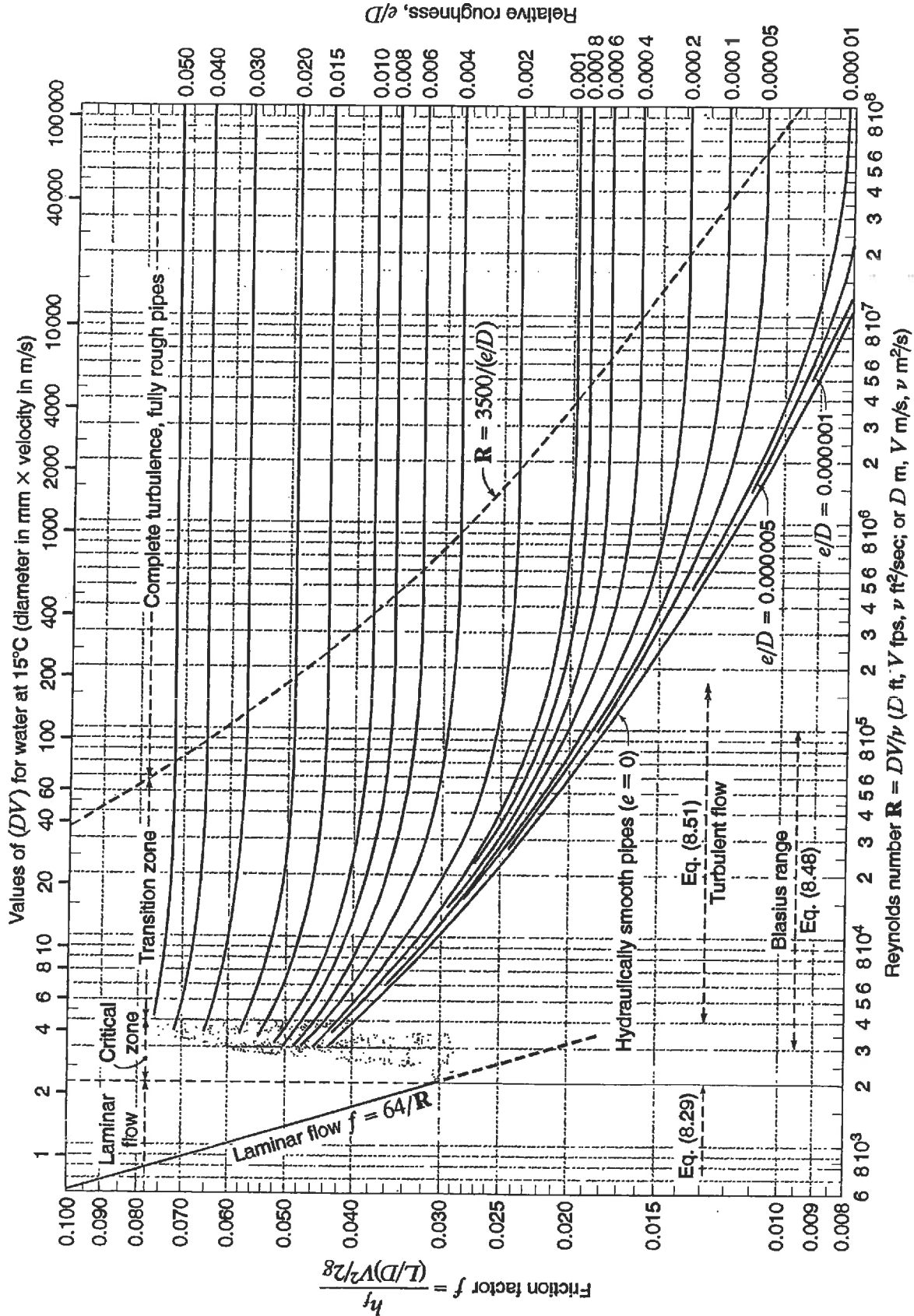**QUESTION 7   MOODY DIAGRAM**   **NAME** ....

Moody chart for pipe friction factor (Stanton diagram).

EXAMINATION PAPER ATTACHMENTS

QUESTION 8   MOODY DIAGRAM   NAME ....

Moody chart for pipe friction factor (Stanton diagram).

QUESTION 9   DRAG DIAGRAM FOR SOLID BODIES   NAME ....

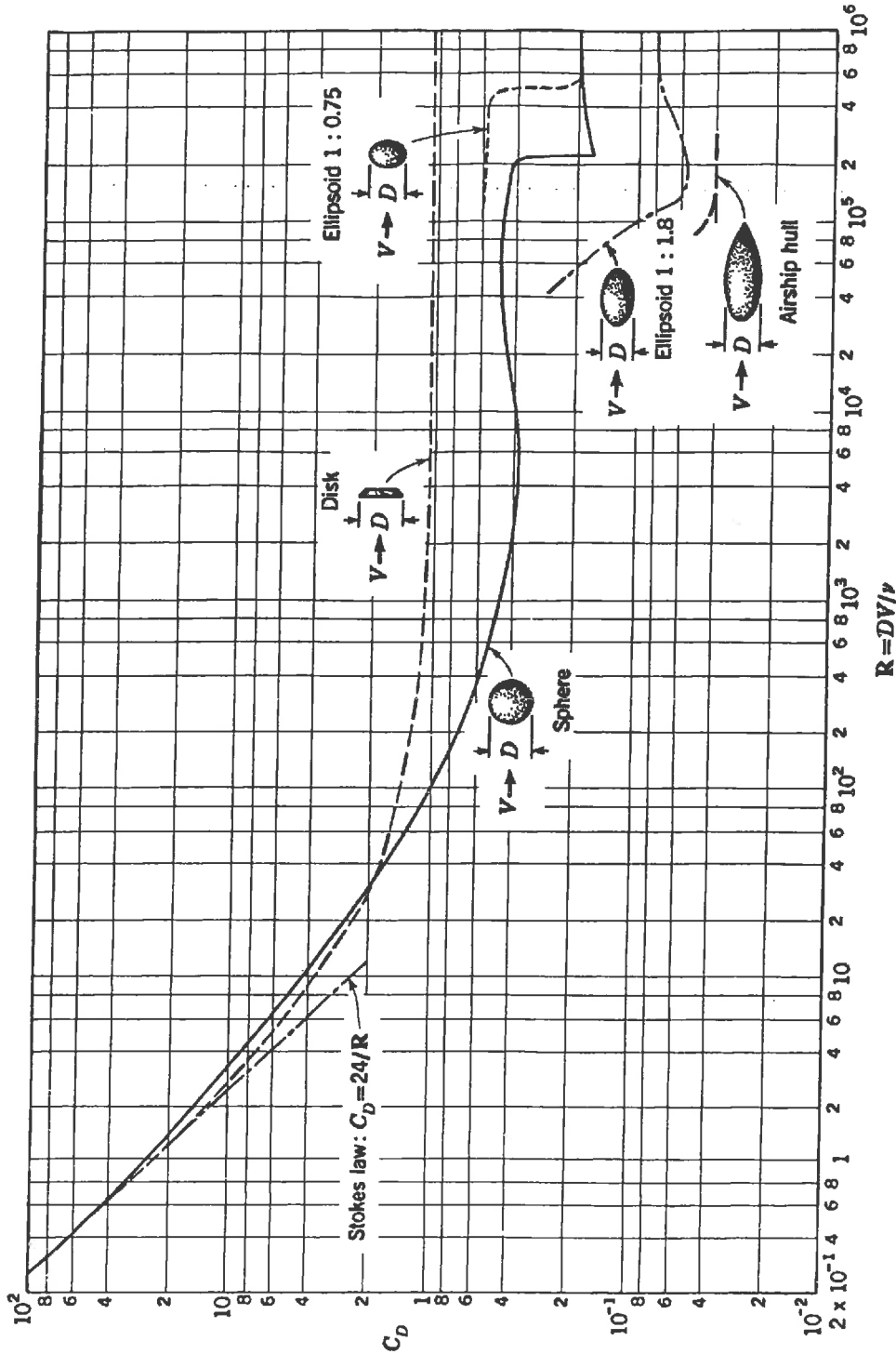**312** *10 Forces on Immersed Bodies*

Figure 10.10 Drag coefficient for bodies of revolution. (Adapted from L. Prandtl, "Ergebnisse der aerodynamischen Versuchsanstalt zu Göttingen," p. 29, R. Oldenbourg, Munich and Berlin, 1923; and F. Eisner, "Das Widerstandsproblem," *Proc. 3d Internatn. Congr. Appl. Mech.,* p. 32, 1930.)
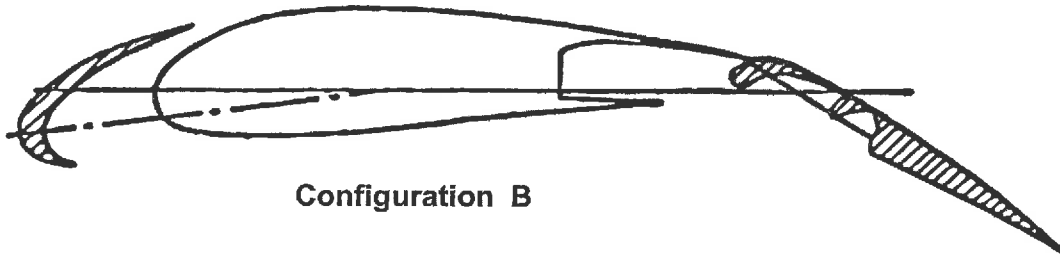
EXAMINATION PAPER ATTACHMENTS

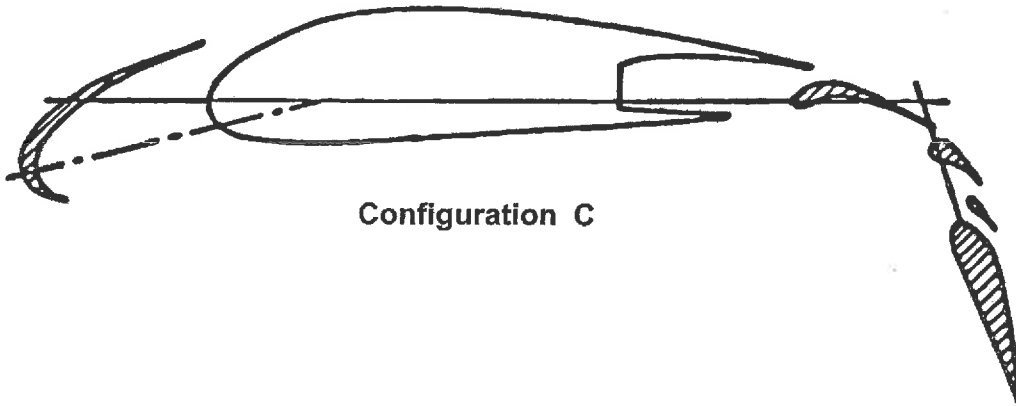**04-BS-7**   NAME ....

QUESTION 12   AIRCRAFT WING

Configuration A

Configuration B

Configuration C

04-BS-7 MECHANICS OF FLUIDS

GENERAL REFERENCE INFORMATION

CONSTANTS

In engineering calculations a high degree of accuracy is seldom attained due to the neglect of minor influences or the inaccuracy of available data.   For consistency in calculations however the following reasonably accurate constants should be used:

Atmospheric Pressure p_o = 100 kPa
Gravitational Acceleration g = 9.81 m/s^2
Specific Gravity of Water = 1.00
Specific Gravity of Glycerine = 1.26
Specific Gravity of Mercury = 13.56
Specific Gravity of Benzene = 0.90
Specific Gravity of Carbon Tetrachloride = 1.59
Density of Water ρ = 1000 kg/m^3
Density of Sea Water ρ = 1025 kg/m^3
Density of Concrete ρ = 2400 kg/m^3
Density of Air ρ = 1.19 kg/m^3 (at 20°C), ρ = 1.21 kg/m^3 (at 15°C)
Absolute Viscosity of Water μ = 1.0 x 10^{-3} Ns/m^2
Absolute Viscosity of Air μ = 1.8 x 10^{-5} Ns/m^2
Surface Tension of Water σ = 0.0728 N/m (at 20°C)
Specific Heat of Water c_p = 4.19 kJ/kg°C
Specific Heat of Air c_p = 1005 J/kg°C
Specific Heat of Air c_p = 718 J/kg°C
Gas Constant for Air R = 287 J/kg°K
Gas Constant for Helium R = 2077 J/kg°K
Gas Constant for Hydrogen R = 4120 J/kg°K

NOMENCLATURE FOR REFERENCE EQUATIONS (SI UNITS)

a	Width	m
A	Flow area, Surface area	m^2
CV	Calorific value	J/kg
c_p	Specific heat at constant pressure	J/kg°C
b	Width	m
D	Diameter	m
E	Energy	J
F	Force	N
g	Gravitational acceleration	m/s^2
h	System head	m
h_L	Head loss	m
H	Pump or turbine head	m
I	Moment of inertia	m^4
k	Ratio of specific heats	
k	Loss coefficient	
K	Constant	
L	Length	m
m	Mass	kg
M	Mass flow rate	kg/s
N	Rotational speed	rev/s
p	Pressure	Pa (N/m^2)
P	Power	W (J/s)
q	Specific heat	J/kg
Q	Flow rate	m^3/s
r	Radius	m
R	Specific gas constant	J/kg K
T	Temperature	K
U	Blade velocity	m/s
v	Specific volume	m^3/kg
V	Velocity	m/s
V	Volume	m^3
w	Specific work	J/kg
W	Work	J
y	Depth	m
z	Elevation	m
η	Efficiency	
μ	Dynamic viscosity	Ns/m^2
ν	Kinematic viscosity	m^2/s
ρ	Density	kg/m^3
σ	Surface tension	N/m
τ	Thrust	N
τ	Shear stress	N/m^2

REFERENCE EQUATIONS

Equation of State

$$p v = R T$$
$$p = \rho R T$$

Universal Gas Law

$$p v^n = constant$$

Compressibility

$$\beta = - \Delta / V \Delta p$$

Viscous Force and Viscosity

$$F = \mu A \, du/dy$$
$$\mu = \tau / (du/dy)$$
$$\nu = \mu / \rho$$

Capillary Rise and Internal Pressure due to Surface Tension

$$h \quad = \quad (\sigma \cos \theta / \rho \, g) \times (perimeter / area)$$
$$p \quad = \quad 2 \sigma / r$$

Pressure at a Point

$$p = \rho g h$$

Forces on Plane Areas and Centre of Pressure

$$F \quad = \quad \rho g y_c A$$
$$y_p \quad = \quad y_c + I_c / y_c A$$

Moments of Inertia

Rectangle: $I_c = b h^3 / 12$
Triangle: $I_c = b h^3 / 36$
Circle: $I_c = \pi D^4 / 64$

Surface Area of Solids

Sphere: $A = \pi D^2$

Volumes of Solids

Sphere: $V = \pi D^3 / 6$
Cone: $V = \pi D^2 h / 12$
Spherical Segment: $V = (3 a^2 + 3 b^2 + 4 h^2) \pi h / 2 g$

Continuity Equation

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = M$$

General Energy Equation

$$p_1 / \rho_1 g + z_1 + V_1^2 / 2 g + q_{in} / g + w_{in} / g$$
$$= p_2 / \rho_2 g + z_2 + V_2^2 / 2 g + h_{L} + q_{out} / g + w_{out} /g$$

Bernoulli Equation

$$p_1 / \rho g + z_1 + V_1^2 / 2 g = p_2 / \rho g + z_2 + V_2^2 / 2 g$$

Momentum Equation

Conduit: $F_R = p_1 A - p_2 A - M (V_2 - V_1)$
Free Jet: $F_R = - \rho Q (V_2 - V_1)$

Flow Measurement

Venturi Tube: $Q = [C A_2 / \{1 - (D_2 / D_1)^4\}^{1/2}][2 g \Delta h]^{1/2}$
Flow Nozzle: $Q = K A_2 [2 g \Delta h]^{1/2}$
Orifice Meter: $Q = K A_o [2 g \Delta h]^{1/2}$

Flow over Weirs

Rectangular Weir: $Q = C_d (2 / 3) [2 g]^{1/2} L H^{3/2}$

Power

Turbomachine: $P = \rho g Q H$
Free Jet: $P = \frac{1}{2} \rho Q V^2$
Moving Blades: $P = M \Delta V U$

Aircraft Propulsion

$F_{thrust} = M (V_{jet} - V_{aircraft})$
$P_{thrust} = M (V_{jet} - V_{aircraft}) V_{aircraft}$
$E_{jet} = \frac{1}{2} (V_{jet}^2 - V_{aircraft}^2)$
$P_{jet} = \frac{1}{2} M (V_{jet}^2 - V_{aircraft}^2)$

$$E_{fuel} = CV_{fuel}$$
$$P_{fuel} = M_{fuel} CV_{fuel}$$
$$\eta_{thermal} = P_{jet} / P_{fuel}$$
$$\eta_{propulsion} = P_{thrust} / P_{jet} = 2 V_{aircraft} / (V_{jet} + V_{aircraft})$$
$$\eta_{overall} = \eta_{thermal} \times \eta_{propulsion}$$

Wind Power

$$P_{total} = \tfrac{1}{2} \rho A_T V_1^3$$
$$P_{max} = 8/27 \, \rho A_T V_1^3$$
$$H_{max} = P_{max} / P_{total} = 16/27$$

Reynolds Number

$$Re = d V \rho / \mu$$

Flow in Pipes

$$h_L = f (L / D) (V^2 / 2 g)$$
$$D_e = 4 \text{ (flow area) / (wetted perimeter)}$$
$$D = D_e \qquad \text{for non-circular pipes}$$
$$L = L_{total} + L_e \qquad \text{for non-linear pipes}$$
$$(L / D) = 35 \, k \qquad \text{for } Re \sim 10^4$$

Drag on Immersed Bodies

Friction Drag: $\quad F_f = C_f \tfrac{1}{2} \rho V^2 B L \quad (B = \pi D)$

Pressure Drag: $\quad F_p = C_p \tfrac{1}{2} \rho V^2 A$

Total Drag: $\quad F_D = C_D \tfrac{1}{2} \rho V^2 A$

Aircraft Wing: $\quad F_L = C_L \tfrac{1}{2} \rho V^2 A_{wing}$

Aircraft Wing: $\quad F_D = C_D \tfrac{1}{2} \rho V^2 A_{wing}$

Karmen Vortex Frequency

$$f \approx 0.20 (V / D) (1 - 20 / Re)$$