National Exams December 2014

04-CHEM-B1, Transport Phenomena

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an OPEN BOOK EXAM.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 points. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

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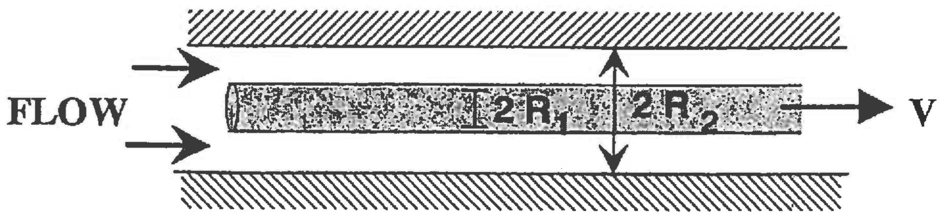
Section A: Fluid Mechanics

A1. The barrel of an extruder can be modeled as if a solid rod is moving with a velocity V through a fluid inside a horizontal cylindrical tube as shown in the figure below.



There is also a pressure gradient imposed on the fluid in the annulus (gap between the solid rod and the cylindrical tube).

a) **[15 points]** Derive an expression for the steady-state velocity distribution for fully developed laminar flow of a Newtonian fluid in the annulus for tube/rod length L.

b) **[10 points]** What is the steady-state velocity distribution for fully developed laminar flow of a Newtonian fluid in the annulus for tube/rod length L if the rod is also stationary?

A2. Dimples on a golf ball cause a drop in the drag force at lower Reynolds number. The table below gives the drag coefficient (C_D) for a rough sphere as a function of Reynolds number (Re).

$Re \times 10^{-4}$	7.5	10	15	20	25
C_D	0.48	0.38	0.22	0.12	0.10

Using a kinematic viscosity (ν) value of 1.69×10^{-4} ft^2/s for air,

a) **[12 points]** Calculate the drag force as a function of velocity for a "dimpled" golf ball of 1.65-inch diameter.

b) **[13 points]** Calculate the drag force for a 1.65-inch "smooth" sphere as a function of velocity, and compare your results with (a).

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Section B: Heat Transfer

B1. The temperatures at the inner and outer surfaces of a plane wall of thickness L are held at the constant valies of T_0 and T_L, where $T_0 > T_L$. The wall material has a thermal conductivity (k) that varies linearly given by the following equation:

$$k = k_0 (1 + \beta T)$$

where k_0 and β are constants.

a) **[15 points]** At what position will the actual temperature profile differ the most from that which would exist in the case of constant thermal conductivity?

b) **[10 points]** Repeat (a) for the case of a hollow cylinder with boundary conditions $T = T_0$ at $r = R_0$ and $T = T_L$ at $r = R_0 + L$.

B2. An apparatus used in a medical operating room to cool blood consists of a coiled tube, which is immersed, in an ice bath. Using this apparatus, blood, flowing at 6 liters/hr, is to be cooled from 40 °C to 30 °C. The inside diameter of the tube is 2.5 mm and the surface coefficient between the ice bath and outside tube surface is 500 W/m2. The thermal resistance of the tube wall may be neglected. Determine the required length of tubing to accomplish the desired cooling using the following properties for blood:

Density (ρ) = 1000 g/cc

Thermal Conductivity (k) = 0.5 W/m.K

Specific Heat Capacity (c_p) = 4.0 kJ/kg.K

Kinematic Viscosity (ν) = 7 x 10^{-7} m^2/s

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Section C: Mass Transfer

C1. One way to deliver a timed dosage of a drug within the human body is to ingest a capsule and allow it to settle in the gastrointestinal system. Once inside the body, the capsule slowly releases the drug to the body by a diffusion-limited process. A suitable drug carrier is a spherical bead of a non-toxic gelatinous material that can pass through the gastrointestinal system without disintegrating. A water-soluble drug (solute A) is uniformly dissolved within the gel, and has an initial concentration (C_{A0}) of 50 mg/cm^3. The drug loaded within the spherical gel capsule is the source of mass transfer, whereas the fluid surrounding the capsule is the sink for mass transfer. Consider a limiting case where the drug is immediately consumed or swept away once it reaches the surface.

a) **[10 points]** Sketch a picture of the physical system and state at least five reasonable assumptions on the mass-transfer aspects of the drug-release process.

b) **[5 points]** What is the simplified differential form of Fick's equation for the drug (species A) within the spherical gel capsule at the surface of the capsule?

c) **[10 points]** What is the simplified form of the general differential equation for mass transfer in terms of concentration C_A? Propose reasonable boundary and initial conditions that may be used to solve the resulting differential equation.

C2. A drop of liquid toluene (A) is kept at a uniform temperature of 25.9 °C, and it is suspended in air (B) by a fine wire at 1 atmosphere total pressure. The initial radius of the drop (r_1) is 2 mm. The vapor pressure of toluene (P_{A1}) at 25.9 °C is 3.84 kPa and the density of liquid toluene (ρ_A) is 866 kg/m^3.

a) **[15 points]** Derive the following equation to predict the time t_f for the drop to evaporate completely in a large volume of still air:

$$t_f = (\rho_A\, r_1^2\, R\, T\, P_{BM})/[2\, M_A\, D_{AB}\, P\, (P_{A1} - P_{A2})]$$

where M_A is the molecular weight of the liquid droplet and P_{BM} is the log mean pressure difference of air.

b) **[10 points]** For $D_{AB} = 8.6 \times 10^{-6}$ m^2/s, calculate the time (in seconds) for complete evaporation of the liquid toluene drop. The molecular weight of toluene is 92.14 g/mol.

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APPENDIX A

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$$\frac{\partial\rho}{\partial t}+(\nabla\cdot\rho\vec{u})=0$$	(1.1)
Rectangular coordinates (x, y, z) $$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u_x)+\frac{\partial}{\partial y}(\rho u_y)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1a)
Cylindrical coordinates (r, θ, z) $$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho u_\theta)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1b)
Spherical coordinates (r, θ, ϕ) $$\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho u_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho u_\phi)=0$$	(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

	$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla P+\vec{g}+\nu(\nabla^2\vec{u})$$	(A2)
Rectangular coordinates (x, y, z)		
x-component	$$\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+g_x+\nu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)$$	(A2a)
y-component	$$\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+g_y+\nu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)$$	(A2b)
z-component	$$\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)$$	(A2c)

Cylindrical coordinates (r, θ, z)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (ru_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right] \quad \text{(A2d)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (ru_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \quad \text{(A2e)}$$

z-component

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \quad \text{(A2f)}$$

Spherical coordinates (r, θ, ϕ)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_r}{\partial \phi}-\frac{u_\theta^2}{r}-\frac{u_\phi^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \phi^2}\right] \quad \text{(A2g)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2}{r}\cot\theta=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \phi^2}\\+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\phi}{\partial \phi}\end{array}\right] \quad \text{(A2h)}$$

ϕ-component

$$\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi}{r}\cot\theta=-\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi}+g_\varphi+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial \phi^2}\\+\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\theta}{\partial \phi}\end{array}\right] \quad \text{(A2i)}$$

Table A.3 The Energy Equation for Incompressible Media

$$\rho c_P\left[\frac{\partial T}{\partial t}+(\vec{u}\cdot\nabla)(T)\right]=[\nabla\cdot k\nabla T]+\dot{T}_G \tag{A3}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \tag{A3a}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \tag{A3b}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right]=$$
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{T}_G \tag{A3c}$$

Table A4: The continuity equation for species *A* in terms of the molar flux

$$\frac{\partial C_A}{\partial t}=-\left(\nabla\cdot\vec{N}_A\right)+\dot{R}_{A,G} \tag{4.}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left(\frac{\partial[N_A]_z}{\partial x}+\frac{\partial[N_A]_y}{\partial y}+\frac{\partial[N_A]_z}{\partial z}\right)+\dot{R}_{A,G} \tag{4a}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r+\frac{1}{r}\frac{\partial}{\partial \theta}[N_A]_\theta+\frac{\partial}{\partial z}[N_A]_z\right\}+\dot{R}_{A,G} \tag{4b}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2[N_A]_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}([N_A]_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}[N_A]_\phi\right\}+\dot{R}_{A,G} \tag{4c}$$

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Table A.5: The continuity equation for species *A*

$$\frac{\partial C_A}{\partial t} + (\vec{u}\cdot\nabla)C_A = D_A\nabla^2 C_A + \dot{R}_{A,G}$$	(5)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial C_A}{\partial t} + u_x\frac{\partial C_A}{\partial x} + u_y\frac{\partial C_A}{\partial y} + u_z\frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x}\left(D\frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$$	(5a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial C_A}{\partial t} + u_r\frac{\partial C_A}{\partial r} + \frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta} + u_z\frac{\partial C_A}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D\frac{\partial C_A}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$$	(5b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial C_A}{\partial t} + u_r\frac{\partial C_A}{\partial r} + \frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D\frac{\partial C_A}{\partial r}\right)$$ $$+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(D\sin\theta\frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial C_A}{\partial \phi}\right) + \dot{R}_{A,G}$$	(5c)