National Exams December 2014

Chem – B3, Simulation, Modeling & Optimization

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumption made.

2. This is a CLOSED BOOK exam, but candidates may use (a) one of two calculators, the CASIO approved models or SHARP approved models, and (b) a scaled (cm or in) ruler.

3. Any five of the seven questions provided constitute a complete paper. If more than five questions are answered, the exam grade will be made up of the five highest marks.

Q1. Evaporation of water from a lean aqueous NaOH solution is to be carried out in an open – top rectangular box – shaped tank with a fixed volume of 500 m^3. Determine the tank dimensions to minimize the cost of material necessary for the construction of the tank. The sides and the bottoms have the same thickness.

Q2. In an unconstrained optimization problem the function $f(x) = x^3 - 7xy + 6y^3$ has to be maximized. (a) Determine if there is, indeed, a maximum, and if yes, at what value of x. (b) Are there other critical (also known as singular) points? If yes, where, and what is their nature?

Q3. The unsteady state fractional concentration distribution of a compound in a narrow and long channel may be expressed as

$$c/c_0 = 1 - \mathrm{erf}\{x/[2(Dt)^{1/2}]\}$$

where c_0 is the constant concentration at the channel inlet (x = 0), D is the dispersion coefficient, and t is time. The error function is defined as

$$\mathrm{erf}(u) \equiv [2/(\pi)^{1/2}] \int_0^u \exp(-z^2)dz$$

Determine the *approximate* value of c/c_0 at x = 31 m and t = 3 month, if D = 3 m^2/month, and if it is known that at x = 30 m and t = 3 month, $c/c_0 = 0.5230$

Q4. The growth of a biochemical culture is described by the empirical discrete model

$$y_{k+2} y_{k+1}^{-3/2} = y_k^{-1/2}$$

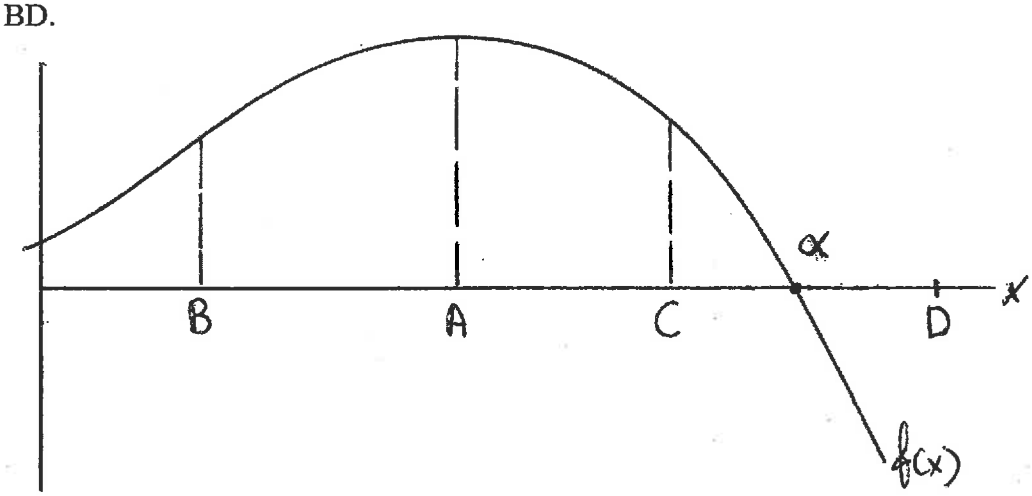
where y_k is the coded concentration at observation instant k. Using the update operator defined as $E(x_k) \equiv x_{k+1}$, $x_k \equiv \log_a(y_k)$ [a: arbitary logarithm base], determine at what (or between what two consecutive) observation instant(s) will the coded concentration reach the numerical value of 85.

Q5. Ten observation pairs were subjected to linear regression $Y = \beta_0 + \beta_1 x$ and the sample regression parameters b_0 = - 0.8701; b_1 = 8.5168 were obtained by least – squares optimization, along with the following parameters: Total sum of squares = 80.61; Total sum due to regression = 76.87; error variance = 0.468; elements of the $\mathbf{C} \equiv (\mathbf{X}^T\mathbf{X})^{-1}$ matrix are: C_{00} = 1.72911; $C_{01} = C_{10}$ = - 1.28981; C_{11} = 0.94354 [the first column of the **X** matrix carries ten elements of numerical value 1, the second column carries the ten independent variable values $x_1, x_2, \ldots, x_{10}$ from top to bottom].(a) Test the hypothesis that Y does not depend on x; (b) Test the hypothesis that regression $Y = \beta_1 x$ can replace the original regression; (c) determine the correlation coefficient (continues on page 2/3).

Critical T – distribution variates pertinent to this problem, at selected degrees of significance, are below.

α	0.50	0.40	0.30	0.20	0.10
$t(\alpha/2)$	0.706	0.889	1.108	1.397	1.860

Q6. Given the function f(x) with a single root α [i.e. $f(\alpha) = 0$] shown in the sketch, determine if the root is reached by (a) Newton's method with starting point x = A; (b) Newton's method with starting point x = B; (c) The bisection (also known as interval halving) method with starting interval $\overline{AD}$; (d) The bisection method with starting interval $\overline{BD}$.



Sketch for Q6.

Q7. A small chemical plant is designed to produce x_1 tonne of product A per year, and x_2 tonne of product B per year. The following parameters have been identified by the designers.

Requirement per year	Product A	Product B
Chemical 1; 180 units available per year	18 units per tonne	3 units per tonne
Chemical 2; 160 units available per year	40 units per tonne	2 units per tonne
Labour time*	24 units per tonne	4 units per tonne

*Agreement with labour unions stipulates that *at least* 120 labour time units per year be consumed

(continues on Page 3/3)

(i) Determine the optimal amounts (tonne per year) of products A and B to be made in the plant to maximize its profit, given that product A sells at 7 MU (monetary unit) per tonne, and product B sells at 4 MU per tonne.

(ii) Sketch in your answer book [no need to plot!] the feasibility region in the $(x_1;x_2)$ plane, and indicate the position of the optimal operating point.

- END OF EXAM -