

NATIONAL EXAMS December 2014
07-Elec-B2 Advanced Control Systems

3 hours duration

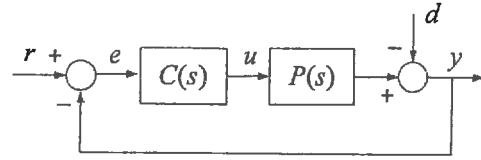
NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use one of two calculators, **a Casio or Sharp approved model.**
3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.
4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.
5. All questions are of equal value.

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1. Consider the control system below with, $P(s) = \frac{16}{(s+2)^2}$, $C(s) = \frac{sK_p + K_i}{s}$

- (a) Design a proportional-integral control system such that the phase margin is at least 45 degrees and the gain crossover is as large as feasible.
- (b) Sketch the magnitude of the Bode plots associated with the closed loop transfer function that relates y to r , and the closed loop transfer function that relates y to d .
- (c) Determine the steady state output, y , for constant inputs, $r = r_0$ and $d = d_0$.



2. Consider the open loop dynamics of a satellite attitude control system,

$$\dot{\theta}(t) = \omega(t)$$

$$\ddot{\theta}(t) = -3\theta(t) - u(t)$$

$$\dot{h}(t) = u(t)$$

Define the state vector, $x(t) = [\theta(t) \ \omega(t) \ h(t)]^T$, the control input, $u(t)$, and the output, $\omega(t)$.

- (a) Determine a state space model for the open loop system.
- (b) Is the system controllable and observable? Justify your answer.
- (c) For the open loop system, let $\theta(0) = 1$, $\dot{\theta}(0) = 0$, $h(0) = 0$, $u(t) = 0$. Determine $\omega(t)$.
- (d) Find a state feedback controller, if it exists, such that the closed loop poles are all located at $s = -2 + j$, $-2 - j$, -4 .

3. The discrete model for a system has the form, $Y(z) = P(z)U(z)$, where, $P(z) = \frac{b}{z-a}$.

- (a) Measurements of $u(k)$ and $y(k)$ are taken at time instants, k , as listed in the Table. Find a least squares estimate for a and b .

k	$y(k)$	$u(k)$
0	300	75
1	387	30
2	375	15
3	324	0

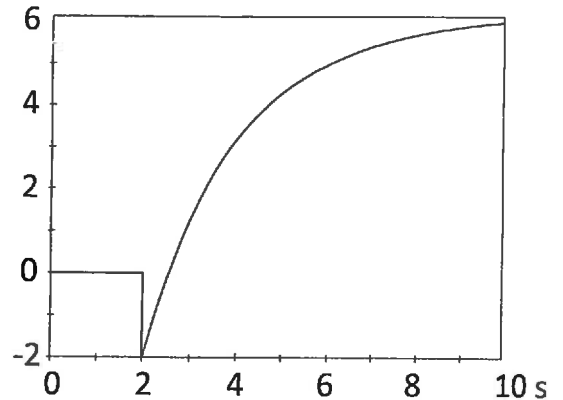
- (b) If $u(k) = 2$, what is the steady state output as predicted by the identified model?

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4. Determine the transfer functions, $P(s)$ and $G(s)$ below.

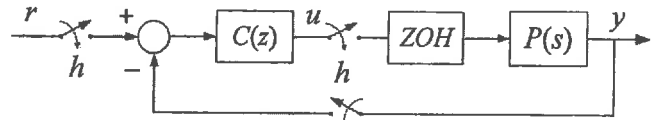
(a) A unit step is applied at the input of an open loop plant, $P(s)$, at time $t = 0$. The measured response is shown on the right. Determine the transfer function, $P(s)$.

(b) When a step of magnitude 2 is applied to the input of a plant, $G(s)$, the steady state output is 10. When a sinusoid of amplitude 2 and frequency 8 rad/sec is applied, the phase lag at the output is 90° and the output amplitude is 15. Assume the system is second order system and has no finite zeros. Find the transfer function, $G(s)$.



5. Consider the sampled data and digital control system below. The input to the ZOH and (continuous) output, y , are uniformly sampled with a sample period of h . $C(z)$ and $P(s)$ are given by,

$$C(z) = Kz^{-1}, \quad P(s) = \frac{1}{3s}$$



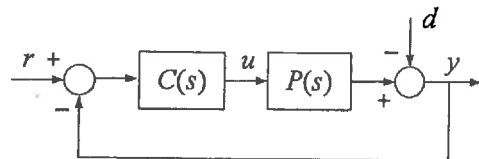
- Determine the discrete closed loop transfer function, $T(z)$, that relates $Y(z)$ to $R(z)$.
- Sketch and annotate the root locus as K varies from zero to infinity.
- Is the closed loop system stable for all values of K ? If not determine the limiting value of K for stability.
- Assume $Kh = 1.5$, and r is initially zero up until $t = 0$, with all initial conditions zero. Suddenly r changes as indicated below.

$$r(0) = 1, r(h) = 0, r(2h) = 0, r(3h) = 0, r(4h) = 0$$

Sketch and carefully annotate the transient response of the continuous output, $y(t)$, for $0 \leq t \leq 4h$.

6. Consider the feedback system below with,

$$C(s) = K, \quad P(z) = \frac{e^{-s}}{8s}$$



- Construct (approximately to scale) the polar or Nyquist plot for the (open) loop transfer function with $K = 1$.
- Determine the gain and phase margin.
- Determine the value of K that results in a phase margin of 50 degrees.
- Determine the steady state value of y when r is a step of magnitude r_0 and d is ramp of slope d_0 .

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Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\frac{A}{s + \alpha}$	$Ae^{-\alpha t}$
$\frac{C + jD}{s + \alpha + j\beta} + \frac{C - jD}{s + \alpha - j\beta}$	$2e^{-\alpha t} (C \cos \beta t + D \sin \beta t)$
$\frac{A}{(s + \alpha)^{n+1}}$	$\frac{At^n e^{-\alpha t}}{n!}$
$\frac{C + jD}{(s + \alpha + j\beta)^{n+1}} + \frac{C - jD}{(s + \alpha - j\beta)^{n+1}}$	$\frac{2t^n e^{-\alpha t}}{n!} (C \cos \beta t + D \sin \beta t)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\frac{Kz}{z - a}$	Ka^n
$\frac{(C + jD)z}{z - re^{j\varphi}} + \frac{(C - jD)z}{z - re^{-j\varphi}}$	$2r^n (C \cos n\varphi - D \sin n\varphi)$
$\frac{Kz}{(z - a)^r}, \quad r = 2, 3, \dots$	$\frac{Kn(n-1)\dots(n-r+2)}{(r-1)! a^{r-1}} a^n$

Table of Laplace and z-Transforms (h denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2 + \beta^2}$	$\frac{z(z - \cos \beta h)}{z^2 - 2z \cos \beta h + 1}$
$\sin \beta t$	$\frac{\beta}{s^2 + \beta^2}$	$\frac{z \sin \beta h}{z^2 - 2z \cos \beta h + 1}$
$e^{-\alpha t} \cos \beta t$	$\frac{s + \alpha}{(s + \alpha)^2 + \beta^2}$	$\frac{z(z - e^{-\alpha h} \cos \beta h)}{z^2 - 2ze^{-\alpha h} \cos \beta h + e^{-2\alpha h}}$
$e^{-\alpha t} \sin \beta t$	$\frac{\beta}{(s + \alpha)^2 + \beta^2}$	$\frac{ze^{-\alpha h} \sin \beta h}{z^2 - 2ze^{-\alpha h} \cos \beta h + e^{-2\alpha h}}$
$t f(t)$	$-\frac{dF(s)}{ds}$	$-zh \frac{dF(z)}{dz}$
$e^{-\alpha t} f(t)$	$F(s + \alpha)$	$F(ze^{\alpha h})$