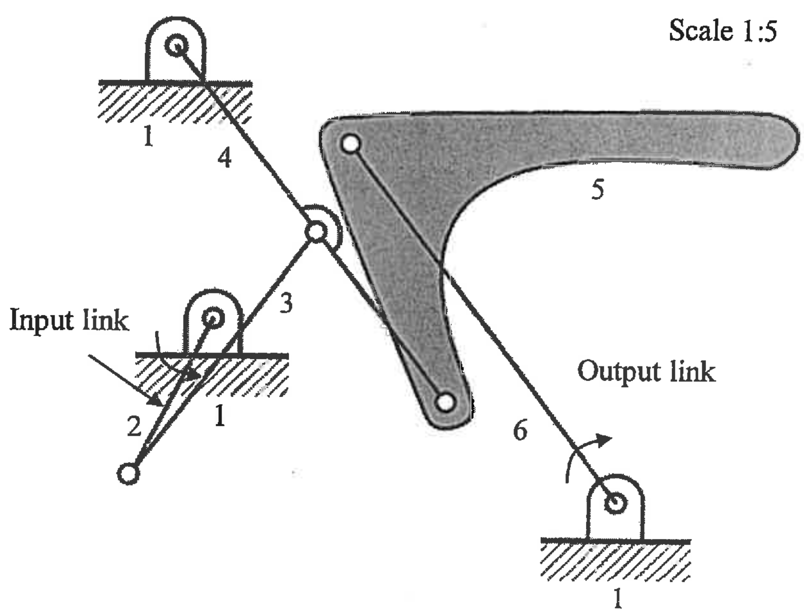
National Exams December 2014
Mec-A2, Kinematics and Dynamics of Machines
3 Hours in Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is an OPEN BOOK exam. Any Sharp or Casio approved calculators are permitted.
3. Answer FIVE questions from the six questions provided.
4. Marks for each question are 20.

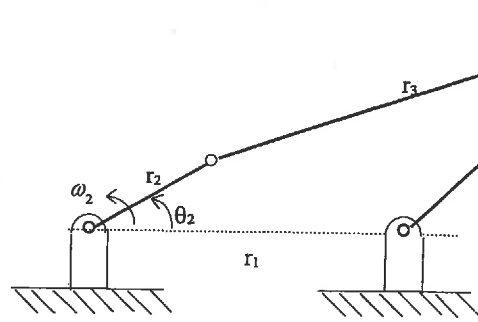
Part A

1. For the six bar mechanism shown below, identify all instant centres on the diagram, and compute the angular velocity of the output link using $I_{2,6}$ if the input link rotates at 1800 rpm.



2. A four-bar function generation mechanism is shown below. The lengths of the base, input, coupler and output links are $r_1 = 0.50$ m, $r_2 = 0.22$ m, $r_3 = 0.69$ m, $r_4 = 0.48$ m, respectively. If the input link rotates at a constant angular velocity, $\omega_2 = 20$ rad / s, determine the angular velocity and angular acceleration of the output link when $\theta_2 = 30°$.

Not to scale.

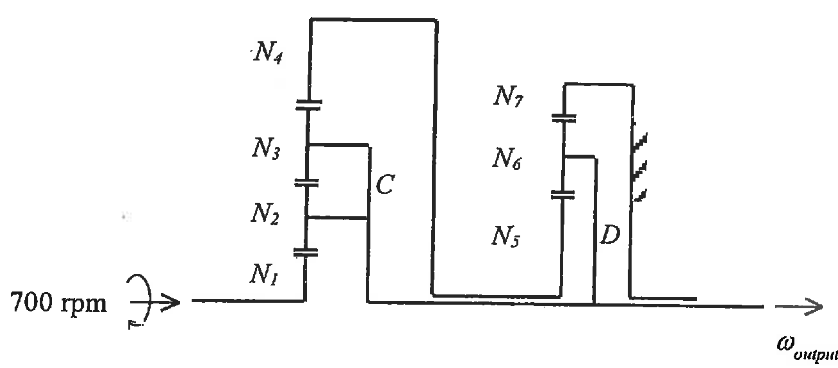


3. For a planetary gear train shown below, gear 7 is fixed; crank arms C and D are attached to the output shaft; gears 4 and 5 form a compound wheel rotating freely about the output shaft. If the numbers of teeth are N1 = 22, N2 = 17, N3 = 20, N4 = 96, N5 = 36, N6 = 18, determine

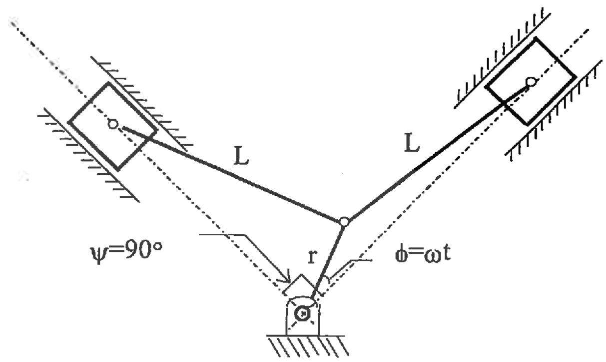
(a) the speed and direction of rotation of the output shaft, and

(b) relative rotational speed of gear 2 with respect to crank arm C.

Not to scale



4. A two-cylinder V-shape engine is located in the same axial plane. Determine, when $\phi = 30^\circ$, the primary resultant shaking force caused by the reciprocating masses, namely, the two pistons of mass 0.5 kg each, as the crank shaft rotates at a constant angular velocity of $\omega = 60$ rad/s.



r=120 mm
L=300 mm

Part B

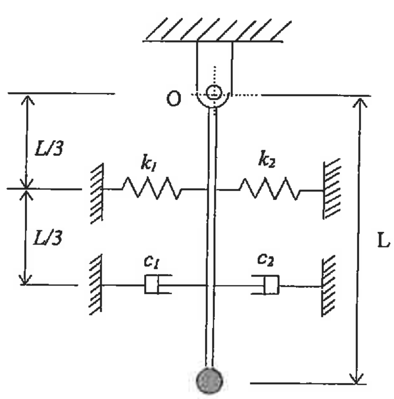
5. A uniform rigid bar of length L and mass m_b rotates about a frictionless hinge O. A concentrated mass m_c is attached to the end of the rigid bar. Two linear springs, k_1 and k_2, and two dashpots, c_1 and c_2, are connected to the same bar at two locations as shown below. The two springs are in their original lengths when the bar is in the vertical position. The effects of the gravitational force must be included in your analysis.

(a) Establish the equation of motion for small amplitude vibration.

(b) Determine the natural frequency of damped vibration.

(c) Determine the free vibration of the system if the pendulum suddenly gains an angular velocity of 2 rad/s (CCW) at the position shown.

$k_1 = k_2 = 100$ N/m, $c_1 = c_2 = 5$ Ns/m, $m_b = 1$ kg, $m_c = 0.5$ kg, $L = 1.2$ m; $g = 9.81$ m/s^2.



6. A lumped vibration system of three degrees of freedom is shown below. For m= 1 kg and k = 10,000 N/m, determine

(a) equations of motion using linear displacements x_1, x_2, and x_3, measured from the static equilibrium positions of the corresponding mass centers, under a harmonic excitation, $F = 10\sin 30t$ (N) applied at mass 1 along coordinate x_1.

(b) modal vectors corresponding to the three natural frequencies $\omega_1 = 36.6$ rad/s, $\omega_2 = 100$ rad/s, $\omega_3 = 136.6$ rad/s

(c) modal matrix $[\Phi]$, normalized to the mass matrix, i.e., the modal matrix satisfying the following orthogonality conditions:

$$[\Phi]^T[M][\Phi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } [\Phi]^T[K][\Phi] = \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ 0 & 0 & \omega_3^2 \end{bmatrix}$$

(d) the steady state response of the system caused by the harmonic excitation.