

National Exams December 2014

98-Ind-B1, Applied Probability & Statistics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a closed book exam. Candidates are permitted to use one of the two permitted calculators (Sharp or Casio models).
3. Candidates are permitted to have an aid sheet consisting of one 8.5" x 11.0" sheet of paper. Writing is permitted on both sides of the paper.
4. This exam consists of four sections (A→D). Within each section, candidates will be given a choice of questions to answer. Please read the instructions for each section carefully. A breakdown of questions and marks is as follows:

Section A:	Do 2 of 3 Questions.	Total marks: 20
Section B:	Do 1 of 2 Questions.	Total marks: 25
Section C:	Do 1 of 2 Questions.	Total marks: 25
Section D:	Do 1 of 2 Questions.	Total marks: 30

Exam:	5 Questions.	Total marks: 100
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4. The value of each question is listed in the exam. Remember to check the instructions for each section. DO NOT ATTEMPT TO DO ALL QUESTIONS.
5. Statistical tables are provided.

Section A: Complete two of the following three questions. This section is worth a total of 20 marks. Do not attempt all questions.

1. Suppose a continuous random variable x has the probability density function $f(x)$ given below:

$$f(x) = \begin{cases} 0 & x \leq 0 \\ \frac{2x}{R^2} & 0 < x < R \\ 0 & \text{otherwise} \end{cases}$$

- Prove that this function is a valid probability density function
- Find the mean of the function
- Find the variance of the function

10 Marks

2. Two random variables have a joint probability density function given by:

$f(x,y)$		x		
		1	2	3
y	1	0.05	0.05	0.1
	2	0.05	0.1	0.35
	3	0	0.2	0.1

- Find the covariance of the random variables X and Y .
- Find the correlation of the random variables X and Y .

10 Marks

3. My neighbours currently have 4 children: 3 girls and 1 boy.
- Determine the probability of a couple with four children having 3 girls and a boy.
 - If my neighbours plan to have another child, what is the probability that the next child born is female?
 - Assume that the number of children a family in Canada has is Poisson distributed with mean 1.5. Determine the probability that a family has 4 or more children

10 Marks

Section B: Complete one of the following two questions. This section is worth a total of 25 marks. Do not attempt all questions.

1. An industrial engineer is conducting a quality experiment on the pull strength (y) of a semi-conductor derived from a particular manufacturing process. 10 samples are randomly selected from the line and subjected to destructive testing. The data is as follows:

Observation	Pull Strength (y)	Wire Length (x_1)	Die Height (x_2)
1	9.95	2	50
2	24.45	8	110
3	31.75	11	120
4	35.00	10	550
5	25.02	8	295
6	16.86	4	200
7	14.38	2	375
8	9.60	2	52
9	24.35	9	100
10	27.50	8	300

- a. Provide the $X'X$ and $X'Y$ matrices. Please note that in some texts these matrices are described as $X^T X$ and $X^T Y$.
- b. Assume that the $(X'X)^{-1}$ matrix is:

0.5509466	-	0.0495021	-	0.0006233
- 0.0495021		0.0095360	-	0.0000536
- 0.0006233	-	0.0000536		0.0000045

Find a linear model of the form $y = b_0 + b_1 x_1 + b_2 x_2$ for this data set.

- c. Complete an ANOVA for the fit model and determine if the regression is significant. You may select any value for α you feel is appropriate, but please justify your choice. To assist in your calculations you may assume SSE is 7.62
- d. Determine which, if any, of the regression coefficients are significant.
- e. How does your answer for part (c) relate to your answer for part (d)?

25 Marks

2. A runner is interested in determining the effect of training and experience on the time required to complete a marathon (i.e. to run 42.2 km). The following table gives the runner's time versus race number:

Race (x)	Time (y)
1	258
2	246
3	236
4	240
5	216
6	212

- Fit a linear regression model of form $y = a + bx$ to this model.
- Determine, using an ANOVA approach, if a linear relationship exists between x and y . Use an α value of 0.05.
- Give a 95% confidence interval for the estimated time of race number 7.
- Provide a 95% prediction interval for the runner's next race (race 7). Explain (briefly, please) the difference between your solutions for c) and d)
- Is a linear model a good fit for this data? Why or why not?

25 Marks

Section C: Complete one of the following two questions. This section is worth a total of 25 marks. Do not attempt all questions.

1. ABX Scientific has developed a new genetically modified hop plant. The plant will, it is believed, grow faster with less susceptibility to disease and with a much greater yield of bitterness. (Hops are traditionally used in beer making to eliminate the sweet taste induced by the use of malted barley.) The unmodified plant typically is known to have a yield of $N(80, 5)$ international bitterness units (IBU) per gram of hops.
 - a. Assume that ABX would consider the modification significant if the IBU of the new plant averaged at least 82 IBU. How large a test sample would be required to ensure that, if the modified plant's IBU reading really was 82, a statistical test of difference would correctly identify the sample as different from the unmodified plant given an α for the statistical test equal to 0.05? You may assume normally distributed values.
 - b. What would you call the calculation performed in part (a)?

Assume that ABX, to test the modified plant, has collected five samples (each of 1 gram) and has determined the IBU in each sample:

Sample	IBU
1	88.5
2	91.0
3	97.7
4	85.0
5	89.7

- c. Assuming that IBUs are normally distributed, use a hypothesis test for a single sample and an $\alpha = 0.05$ to test the following:

$$H_0: \mu = 85$$

$$H_1: \mu > 85$$

- d. Calculate a 90% Confidence Interval for the mean of the sample in part c.
 - e. Calculate a 90% Confidence Interval for the sample variance in part c.

25 Marks

2. Hermes Sports is a Victoria area sports manufacturer. The firm has recently invented a new "smart shoe" that uses small microprocessors to vary the stiffness and pitch of the shoe as an individual runs. Such adjustments are particularly effective for people running long distance races where the foot geometry and length can change noticeably over the course of a run.

To test the new shoe, the firm recruited 20 individuals of varying running abilities in the Lower Mainland area of BC. 10 subjects raced a marathon in Vancouver wearing regular Hermes shoes and 10 subjects ran a marathon in Victoria wearing the new smart shoes.

The results from the races are as follows:

Vancouver	Victoria
164.20	166.15
212.69	195.49
157.75	141.97
237.43	238.87
223.75	225.77
222.39	218.82
162.54	163.31
166.89	164.96
159.79	153.46
197.26	178.69

- Determine, using $\alpha = 0.05$, whether the variances between the two conditions are the same.
- Determine, using $\alpha = 0.05$, whether the new smart shoe provides a better result (faster time) than the regular shoe. State any assumptions you make in answering this question.
- Provide a 95% confidence interval for the difference between the finishing times for the two samples.
- Relate the results of part b and part c. How could you derive the answer for part b from the results of part c? Note here that I am not looking for any calculations, but a short description.
- Is this a good way to test the effectiveness of the new shoe? Is there any potential for bias induced by the test design? What suggestions might you have (be brief please) to improve the design of the test?

25 Marks

Section D: Complete one of the following two questions. This section is worth a total of 30 marks. Do not attempt all questions.

1. An Industrial Engineer is conducting an experiment in which the throughput of a manufacturing line is being evaluated. The line is being run at 4 different speeds (0.5 m/s, 0.6 m/s, 0.7m/s and 0.8 m/s) and two different policies for inspecting and repairing defects (each operator repairs versus a dedicated repair person at the end of the line). Throughput is measured in terms of completed parts per minute:

		Line Speed			
		0.5	0.6	0.7	0.8
Repair	Individual	2.3	3.4	3.8	3.9
		2.9	3.7	3.9	3.2
		3.1	3.6	4.1	3.0
		3.2	3.2	3.8	2.7
	Dedicated	4.3	3.8	3.9	3.5
		3.9	3.8	4.0	3.6
		3.9	3.9	3.7	3.8
		4.2	3.5	3.6	3.9

Perform an analysis of variance and determine ($\alpha = 0.05$):

- Does line speed have a significant impact on throughput?
- Does repair policy have a significant impact on throughput?
- Is there an interaction between the two factors?
- Interpret your answer to (c) and identify how the IE should find an optimal setting for these two variables to maximize throughput, if possible.
- Calculate R^2 .
- Is this a good model? Why or why not?
- What assumptions underlie your ANOVA model?

To assist you in your calculations, you may assume that $SST = 6.25$ and that $SS(AB) = 1.893$. Similarly, you may assume that the grand sum is 115.1 and the grand mean is 3.60.

30 Marks

2. An IE is evaluating the work methods at four workstations in a plant that processes hops (a type of coniferous plant that is used in the brewing of beer). Each of the four workstations involves packing (i.e. gathering the hops into a Kevlar bag, weighing, and sealing). Each of the operators has been issued with the same standard procedure for packing. However, management feels that the resulting packing times are not the same. A sample of three cycles was collected at each workstation. The resulting cycle times (in seconds) are listed below:

WS 1	WS 2	WS 3	WS 4
25.6	25.2	20.8	31.6
24.3	28.6	26.7	29.8
27.9	24.7	22.2	34.3
26.2	26.1	24.5	31.2

- Perform an analysis of variance at $\alpha = 0.05$ level of significance.
- Use Bartlett's Test to determine if the variances of the four samples are equal. Assume an $\alpha = 0.01$. Relate your answer to that of part (a). What does this test say about your analysis?
- Determine, using Tukey's Test, whether any of the workstations differ significantly. Assume a significance level of 0.05. Relate your answer to that of part (a). What does this test say about your analysis?

30 Marks

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.00	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Critical Values of the t-Distribution

v	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
50	0.255	0.528	0.849	1.047	1.299	1.676	2.009
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
70	0.254	0.527	0.847	1.044	1.294	1.667	1.994
80	0.254	0.526	0.846	1.043	1.292	1.664	1.990
90	0.254	0.526	0.846	1.042	1.291	1.662	1.987
100	0.254	0.526	0.845	1.042	1.290	1.660	1.984
110	0.254	0.526	0.845	1.041	1.289	1.659	1.982
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
130	0.254	0.526	0.844	1.041	1.288	1.657	1.978
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Critical Values of the t-Distribution

v	α						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.895	21.205	31.821	42.433	63.657	127.321	636.619
2	4.849	5.643	6.965	8.073	9.925	14.089	31.599
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
50	2.109	2.234	2.403	2.519	2.678	2.937	3.496
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
70	2.093	2.215	2.381	2.494	2.648	2.899	3.435
80	2.088	2.209	2.374	2.486	2.639	2.887	3.416
90	2.084	2.205	2.368	2.480	2.632	2.878	3.402
100	2.081	2.201	2.364	2.475	2.626	2.871	3.390
110	2.078	2.199	2.361	2.471	2.621	2.865	3.381
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
130	2.075	2.194	2.355	2.465	2.614	2.856	3.367
∞	2.054	2.170	2.327	2.433	2.576	2.808	3.291

Critical Values of the Chi-Squared Distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.000	0.000	0.001	0.001	0.004	0.016	0.064	0.102	0.148	0.455
2	0.010	0.020	0.040	0.051	0.103	0.211	0.446	0.575	0.713	1.386
3	0.072	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336

Critical Values of the Chi-Squared Distribution

v	α									
	0.3	0.25	0.2	0.1	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.828
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.816
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.467
5	6.064	6.626	7.289	9.236	11.070	12.833	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.458
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.559	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703

Critical Values of the F Distribution

		$f_{0.05(v_1, v_2)}$									
v2	v1										
	1	2	3	4	5	6	7	8	9	10	
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	

Critical Values of the F Distribution

		$f_{0.05(v1,v2)}$									
v2	v1										
	10	12	15	20	24	30	40	60	120	∞	
1	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.30	
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37	
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.41	
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67	
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65	
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64	
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
50	2.03	1.95	1.87	1.78	1.74	1.69	1.63	1.58	1.51	1.44	
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.26	
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.01	

Critical Values of the F Distribution

		$f_{0.01}(v_1, v_2)$									
v2	v1										
	1	2	3	4	5	6	7	8	9	10	
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	

Critical Values of the F Distribution

		$f_{0.01(v1, v2)}$									
v2	v1										
	10	12	15	20	24	30	40	60	120	∞	
1	6055.85	6106.32	6157.28	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.55	
2	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50	
3	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13	
4	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46	
5	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02	
6	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88	
7	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65	
8	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86	
9	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31	
10	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91	
11	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60	
12	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36	
13	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17	
14	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.01	
15	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87	
16	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75	
17	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65	
18	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57	
19	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49	
20	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42	
21	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36	
22	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31	
23	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26	
24	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21	
25	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17	
26	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13	
27	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10	
28	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.07	
29	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.04	
30	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01	
40	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.81	
50	2.70	2.56	2.42	2.27	2.18	2.10	2.01	1.91	1.80	1.68	
60	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60	
120	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38	
∞	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.01	

Critical Values for Bartlett's Test

$b_k(0.01;n)$									
Number of populations, k									
n	2	3	4	5	6	7	8	9	10
3	0.1411	0.1672							
4	0.2843	0.3165	0.3475	0.3729	0.3937	0.4110			
5	0.3984	0.4304	0.4607	0.4850	0.5046	0.5207	0.5343	0.5458	0.5558
6	0.4850	0.5149	0.5430	0.5653	0.5832	0.5975	0.6100	0.6204	0.6293
7	0.5512	0.5787	0.6045	0.6248	0.6410	0.6542	0.6652	0.6744	0.6824
8	0.6031	0.6282	0.6518	0.6704	0.6851	0.6970	0.7069	0.7153	0.7225
9	0.6445	0.6676	0.6892	0.7062	0.7197	0.7305	0.7395	0.7471	0.7536
10	0.6783	0.6996	0.7195	0.7352	0.7475	0.7575	0.7657	0.7726	0.7786
11	0.7063	0.7260	0.7445	0.7590	0.7703	0.7795	0.7871	0.7935	0.7990
12	0.7299	0.7483	0.7654	0.7789	0.7894	0.7980	0.8050	0.8109	0.8160
13	0.7501	0.7672	0.7832	0.7958	0.8056	0.8135	0.8201	0.8256	0.8303
14	0.7674	0.7835	0.7985	0.8103	0.8195	0.8269	0.8330	0.8382	0.8426
15	0.7825	0.7977	0.8118	0.8229	0.8315	0.8385	0.8443	0.8491	0.8532
16	0.7958	0.8101	0.8235	0.8339	0.8421	0.8486	0.8541	0.8586	0.8625
17	0.8076	0.8211	0.8338	0.8436	0.8514	0.8576	0.8627	0.8670	0.8707
18	0.8181	0.8309	0.8429	0.8523	0.8596	0.8655	0.8704	0.8745	0.8780
19	0.8275	0.8397	0.8512	0.8601	0.8670	0.8727	0.8773	0.8811	0.8845
20	0.8360	0.8476	0.8586	0.8671	0.8737	0.8791	0.8835	0.8871	0.8903

Upper Percentage Points of the Studentized Range Distribution: Values of $q(0.05, k, \nu)$

Degrees of Freedom ν	Number of Treatments k									
	2	3	4	5	6	7	8	9	10	
1	18.00	27.00	32.80	37.20	40.50	43.10	45.10	47.10	49.10	
2	6.09	5.33	9.80	10.89	11.73	12.43	13.03	13.54	13.99	
3	4.50	5.91	6.83	7.51	8.04	8.47	8.85	9.18	9.46	
4	3.93	5.04	5.76	6.29	6.71	7.06	7.35	7.60	7.83	
5	3.64	4.60	5.22	5.67	6.09	6.80	6.58	6.33	6.03	
6	3.46	4.34	4.90	5.30	6.49	6.32	6.12	5.90	5.63	
7	3.34	4.16	4.68	5.06	6.16	6.00	5.82	5.61	5.36	
8	3.26	4.04	4.53	4.89	5.92	5.77	5.60	5.40	5.17	
9	3.20	3.95	4.41	4.76	5.74	5.59	5.43	5.24	5.02	
10	3.15	3.88	4.33	4.65	5.60	5.46	5.30	5.12	4.91	
11	3.11	3.82	4.26	4.57	5.49	5.35	5.20	5.03	4.82	
12	3.08	3.77	4.20	4.51	5.39	5.27	5.12	4.95	4.75	
13	3.06	3.73	4.15	4.45	5.32	5.19	5.05	4.88	4.69	
14	3.03	3.70	4.11	4.41	5.25	5.13	4.99	4.83	4.64	
15	3.01	3.67	4.08	4.37	5.20	5.08	4.94	4.78	4.59	
16	3.00	3.65	4.05	4.33	5.15	5.03	4.90	4.74	4.56	
17	2.98	3.63	4.02	4.30	5.11	4.99	4.86	4.70	4.52	
18	2.97	3.61	4.00	4.28	5.07	4.96	4.82	4.67	4.49	
19	2.96	3.59	3.98	4.25	5.04	4.92	4.79	4.65	4.47	
20	2.95	3.58	3.96	4.23	5.01	4.90	4.77	4.62	4.45	
24	2.92	3.53	3.90	4.17	4.92	4.81	4.68	4.54	4.37	
30	2.89	3.49	3.85	4.10	4.82	4.72	4.60	4.46	4.30	
40	2.86	3.44	3.79	4.04	4.73	4.63	4.52	4.39	4.23	
60	2.83	3.40	3.74	3.98	4.65	4.55	4.44	4.31	4.16	
120	2.80	3.36	3.68	3.92	4.56	4.47	4.36	4.24	4.10	
∞	2.77	3.31	3.63	3.86	4.47	4.39	4.29	4.17	4.03	