

National Exams May 2014
04-CHEM-B1, Transport Phenomena
3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an OPEN BOOK EXAM.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 marks. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

National Exams
04-CHEM-B1, Transport Phenomena

Section A: Fluid Mechanics

A1. [25 marks overall] Consider the laminar flow of water between two infinite parallel plates that are spaced 2δ apart.

- a) **[15 marks]** Starting with the appropriate form of the conservation equation (see Appendix A), show that the steady-state velocity profile between the plates is given by:

$$u = u_{\max} \left(1 - \left(\frac{y}{\delta} \right)^2 \right)$$

- b) **[10 marks]** If the inlet temperature of the water is T_{in} and the surfaces of the plates are maintained at a temperature of T_s , again using the appropriate conservation equation derive the differential equation that describes the steady-state temperature profile. **Do not attempt to solve this equation, but state all appropriate boundary conditions.**

A2. [25 marks] Consider the spinning cone viscometer shown in Fig. A1. You may assume that the velocity profile between the spinning cone and the base is linear. The cone angle is 45° and the cone is spinning at 1500 rpm. Determine the torque required to keep the cone spinning if the viscosity of the fluid is $1.5 \text{ Pa}\cdot\text{s}$.

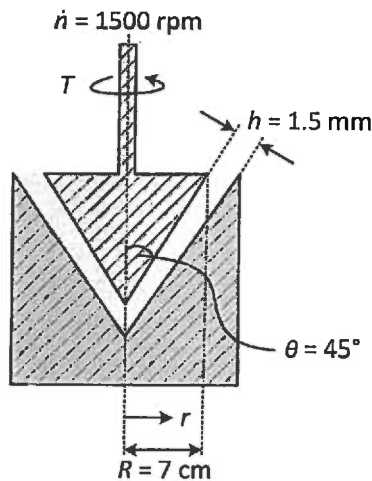


Fig. A1: Spinning cone viscometer

National Exams
04-CHEM-B1, Transport Phenomena

Section B: Heat Transfer

- B1. [25 marks overall]** Heat is generated within a hollow cylinder as shown in Fig. B1 at a rate of 2000 W per unit length. The inner surface of the cylinder is insulated and has a temperature of T_1 . The outer surface is exposed to air at a temperature of $T_\infty = 25^\circ\text{C}$ and a heat transfer coefficient of $h = 75 \text{ W m}^{-2} \text{ K}^{-1}$. The thermal conductivity of the cylinder is $k = 1.2 \text{ W m}^{-1} \text{ K}^{-1}$ and the inner and outer radii are 0.02 m and 0.04 m, respectively.

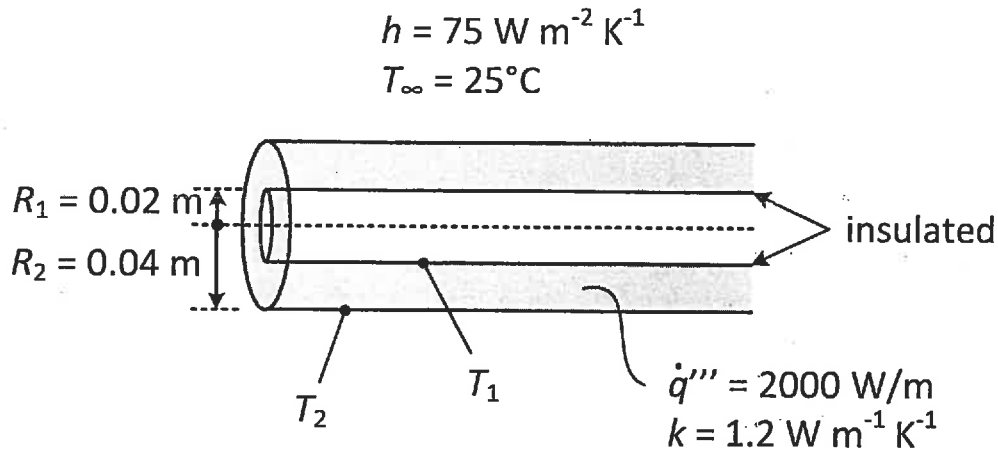


Fig. B1: heat generation in a hollow cylinder

- a) [20 Marks] Starting with the appropriate conservation equation (see Appendix A), show that the temperature profile in the hollow cylinder is:
- $$T = \frac{\dot{q}'''}{2} \left[\frac{1}{k} \left(R_1^2 \ln\left(\frac{r}{R_2}\right) + \frac{R_2^2 - r^2}{2} \right) - \frac{1}{h} \left(\frac{R_1^2}{R_2} - R_2 \right) \right] + T_\infty$$
- b) [5 marks] What is the maximum temperature in the cylinder?
- B2. [25 marks overall]** A square composite wall (20 cm by 20 cm) is insulated on five sides as shown in Fig B2 (note that the front and back are also insulated). A very thin square heater (negligible thickness) is sandwiched between the copper and alumina sections. The heater delivers 500 W of heat uniformly across its cross-section. Both convection and radiation heat transfer occurs on the open face of the wall. The surface temperature of the open wall is 55°C at steady-state. Use the information given on the diagram to determine:
- a) [10 Marks] The temperature of the thin heater (T_2).
- b) [5 Marks] The temperature at the copper-insulation interface (T_1).
- c) [10 Marks] The convective heat transfer coefficient, h .

National Exams
04-CHEM-B1, Transport Phenomena

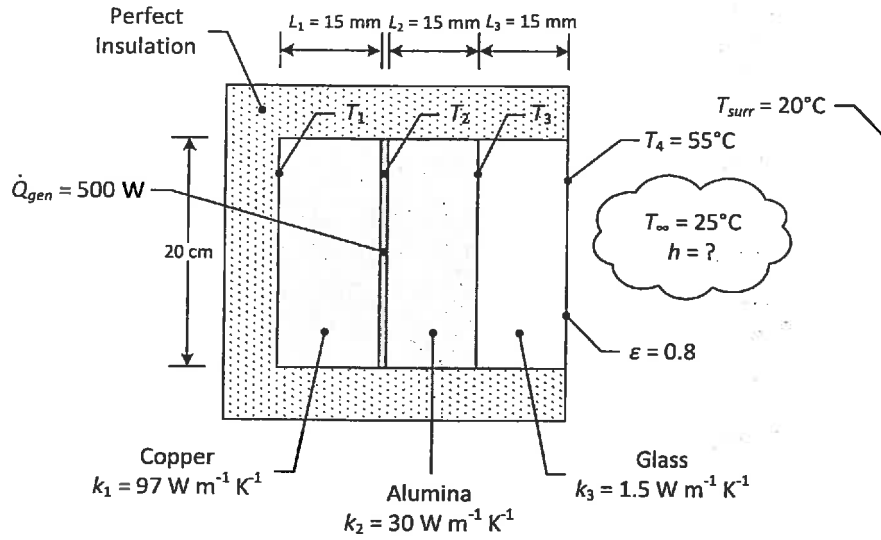


Fig. B2: Composite wall

National Exams
04-CHEM-B1, Transport Phenomena

Section C: Mass Transfer

- C1. [25 marks] As a gas (component A) diffuses to a solid catalyst surface as shown in Fig. C1, the reaction $A_{(g)} \rightarrow B_{(g)}$ occurs at a constant volumetric rate $-k_A$. At the catalyst surface, the instantaneous reaction $A_{(g)} \rightarrow B_{(g)}$ occurs. As component B diffuses back through the boundary layer, component B reacts to form another gas (component C) according to $B_{(g)} \rightarrow C_{(g)}$ at a constant volumetric rate $-k_B$.

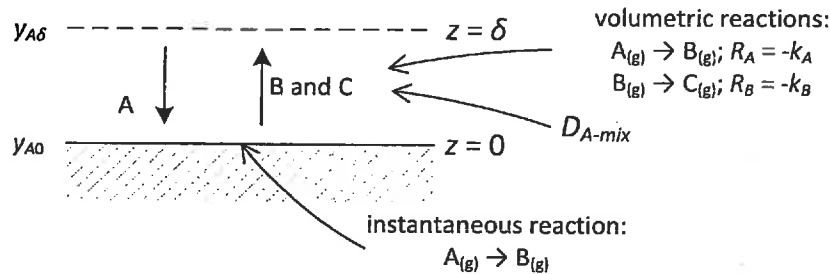


Fig. C1: Gas diffusing to a solid surface

Starting with the appropriate conservation equation (see Appendix A), show that the mole fraction profile of A in the boundary layer is,

$$y_A = \frac{z}{\delta} y_{A\delta} - \frac{k_A (\delta z - z^2)}{2cD_{A-mix}}$$

and that the flux of component A through the boundary layer at any location is given by,

$$N_A = k_A \left(\frac{\delta}{2} - z \right) - \frac{cD_{A-mix}}{\delta} y_{A\delta}$$

- C2. [25 marks overall] Helium gas at 25°C and 4 bar is contained in a glass tube as shown in Fig. C2. The inner and outer diameters of the tube are 100 mm and 110 mm, respectively. At these conditions, the concentration of helium at the inside surface of the glass is 1200 mol/m³. The helium concentration on the outer surface of the glass can be assumed to be negligible. Using a mass diffusivity of helium in the glass of 2.3×10^{-10} m²/s, determine:

- [15 Marks] The rate at which helium diffuses through the glass per unit length of tube.
- [10 Marks] The helium concentration halfway between the inner and outer surface of the glass tube (at $r = 52.5$ mm).

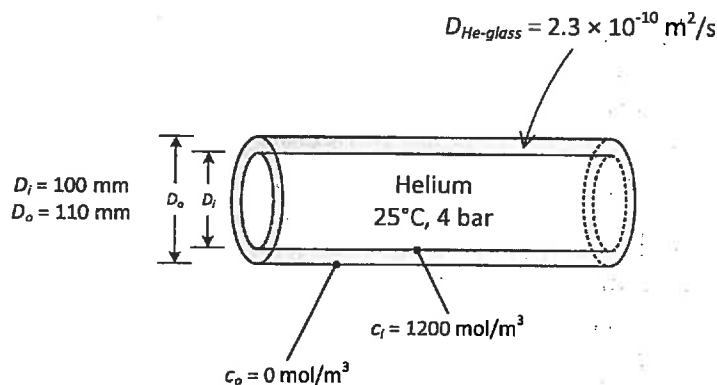


Fig. C2: Glass tube containing He

National Exams
04-CHEM-B1, Transport Phenomena

APPENDIX A

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$\frac{\partial \rho}{\partial t} + (\nabla \cdot \rho \vec{u}) = 0 \quad (1.1)$
<p>Rectangular coordinates (x, y, z)</p> $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0 \quad (1.1a)$
<p>Cylindrical coordinates (r, θ, z)</p> $\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0 \quad (1.1b)$
<p>Spherical coordinates (r, θ, φ)</p> $\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(\rho r^2 u_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(\rho u_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}(\rho u_\phi) = 0 \quad (1.1c)$

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\frac{1}{\rho} \nabla P + \vec{g} + \nu (\nabla^2 \vec{u}) \quad (A2)$
<p>Rectangular coordinates (x, y, z)</p>
<p>x-component</p> $\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + g_x + \nu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) \quad (A2a)$
<p>y-component</p> $\frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + g_y + \nu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) \quad (A2b)$
<p>z-component</p> $\frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \nu \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right) \quad (A2c)$

National Exams
04-CHEM-B1, Transport Phenomena

Cylindrical coordinates (r, θ, z)

$$\begin{aligned} & \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} \\ r\text{-component} \quad & = -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_r + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (ru_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right] \end{aligned} \quad (\text{A2d})$$

$$\begin{aligned} & \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} \\ \theta\text{-component} \quad & = -\frac{1}{\rho r} \frac{\partial P}{\partial \theta} + g_\theta + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (ru_\theta)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2} \right] \end{aligned} \quad (\text{A2e})$$

$$\begin{aligned} & \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \\ z\text{-component} \quad & = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \nu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] \end{aligned} \quad (\text{A2f})$$

Spherical coordinates (r, θ, ϕ)

$$\begin{aligned} & \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + \left(\frac{u_\phi}{r \sin \theta} \right) \frac{\partial u_r}{\partial \phi} - \frac{u_\theta^2}{r} - \frac{u_\phi^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_r \\ r\text{-component} \quad & + \nu \left[\frac{1}{r^2} \frac{\partial^2}{\partial r^2} (r^2 u_r) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u_r}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_r}{\partial \phi^2} \right] \end{aligned} \quad (\text{A2g})$$

$$\begin{aligned} & \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \left(\frac{u_\phi}{r \sin \theta} \right) \frac{\partial u_\theta}{\partial \phi} + \frac{u_r u_\theta}{r} - \frac{u_\phi^2}{r} \cot \theta = -\frac{1}{\rho r} \frac{\partial P}{\partial \theta} + g_\theta \\ \theta\text{-component} \quad & + \nu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (u_\theta \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\theta}{\partial \phi^2} \right. \\ & \left. + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial u_\phi}{\partial \phi} \right] \end{aligned} \quad (\text{A2h})$$

$$\begin{aligned} & \frac{\partial u_\phi}{\partial t} + u_r \frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r u_\phi}{r} + \frac{u_\theta u_\phi}{r} \cot \theta = -\frac{1}{\rho r \sin \theta} \frac{\partial P}{\partial \phi} \\ \phi\text{-component} \quad & + g_\phi + \nu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (u_\phi \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\phi}{\partial \phi^2} \right. \\ & \left. + \frac{2}{r^2 \sin \theta} \frac{\partial u_r}{\partial \phi} + \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial u_\theta}{\partial \phi} \right] \end{aligned} \quad (\text{A2i})$$

National Exams
04-CHEM-B1, Transport Phenomena

Table A.3 The Energy Equation for Incompressible Media

$\rho c_p \left[\frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla)(T) \right] = [\nabla \cdot k \nabla T] + \dot{T}_G \quad (A3)$
<p>Rectangular coordinates (x, y, z)</p> $\rho c_p \left[\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} \right] = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{T}_G \quad (A3a)$
<p>Cylindrical coordinates (r, θ, z)</p> $\rho c_p \left[\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z} \right] = \frac{1}{r} \frac{\partial}{\partial r} \left(r k \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(k \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{T}_G \quad (A3b)$
<p>Spherical coordinates (r, θ, φ)</p> $\rho c_p \left[\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right] =$ $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \dot{T}_G \quad (A3c)$

Table A4: The continuity equation for species A in terms of the molar flux

$\frac{\partial C_A}{\partial t} = -(\nabla \cdot \vec{N}_A) + \dot{R}_{A,G} \quad (4.)$
<p>Rectangular coordinates (x, y, z)</p> $\frac{\partial C_A}{\partial t} = - \left(\frac{\partial [N_A]_x}{\partial x} + \frac{\partial [N_A]_y}{\partial y} + \frac{\partial [N_A]_z}{\partial z} \right) + \dot{R}_{A,G} \quad (4a)$
<p>Cylindrical coordinates (r, θ, z)</p> $\frac{\partial C_A}{\partial t} = - \left\{ \frac{1}{r} \frac{\partial}{\partial r} [r N_A]_r + \frac{1}{r} \frac{\partial}{\partial \theta} [N_A]_\theta + \frac{\partial}{\partial z} [N_A]_z \right\} + \dot{R}_{A,G} \quad (4b)$
<p>Spherical coordinates (r, θ, φ)</p> $\frac{\partial C_A}{\partial t} = - \left\{ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 [N_A]_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} ([N_A]_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} [N_A]_\phi \right\} + \dot{R}_{A,G} \quad (4c)$

National Exams
04-CHEM-B1, Transport Phenomena

Table A.5: The continuity equation for species A

$\frac{\partial C_A}{\partial t} + (\vec{u} \cdot \nabla) C_A = D_A \nabla^2 C_A + \dot{R}_{A,G} \quad (5)$
<p>Rectangular coordinates (x, y, z)</p> $\frac{\partial C_A}{\partial t} + u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x} \left(D \frac{\partial C_A}{\partial x} \right) + \frac{\partial}{\partial y} \left(D \frac{\partial C_A}{\partial y} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial C_A}{\partial z} \right) + \dot{R}_{A,G} \quad (5a)$
<p>Cylindrical coordinates (r, θ, z)</p> $\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + u_z \frac{\partial C_A}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(D \frac{\partial C_A}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial C_A}{\partial z} \right) + \dot{R}_{A,G} \quad (5b)$
<p>Spherical coordinates (r, θ, φ)</p> $\begin{aligned} \frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial C_A}{\partial \phi} &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial C_A}{\partial r} \right) \\ + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(D \sin \theta \frac{\partial C_A}{\partial \theta} \right) &+ \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(D \frac{\partial C_A}{\partial \phi} \right) + \dot{R}_{A,G} \end{aligned} \quad (5c)$