PROFESSIONAL ENGINEERS OF ONTARIO

ANNUAL EXAMINATIONS – December 2015

07-Mec-B10 Finite Element Analysis

3 hours duration

INSTRUCTIONS:

1. **If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) made with the answer.**

2. **This examination paper is open book; candidates are permitted to make use of any textbooks, references or notes.**

3. **Any non-communicating calculator is permitted. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.**

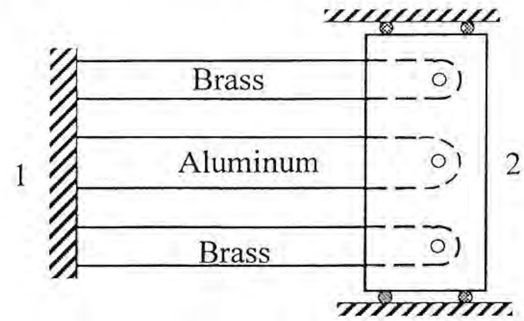
4. Candidates are required to attempt any five questions. **The questions are to be solved within the context of the finite element method.**

5. **The questions are equally weighted. Indicate which five questions are to be marked on the cover of the first examination workbook.**

Question 1. [20 marks] A shown bar assemblage comprises two outer brass bars and an inner aluminum bar. The three-bar assemblage is subjected to a temperature drop of 20°C. Assume the lengths of each bar $L = 2.5$ m. For the aluminum bar, take $E_{alum} = 70$ GPa, $\alpha_{alum} = 23 \times 10^{-6}$ (mm/mm)/°C, and cross-sectional area $A_{alum} = 10 \times 10^{-4}$ m^2. For the brass bars, take $E_{brass} = 100$ GPa, $\alpha_{brass} = 20 \times 10^{-6}$ (mm/mm)/°C, and cross-sectional area $A_{brass} = 5 \times 10^{-4}$ m^2. Determine

(a) [14 marks] the displacement of node 2 and

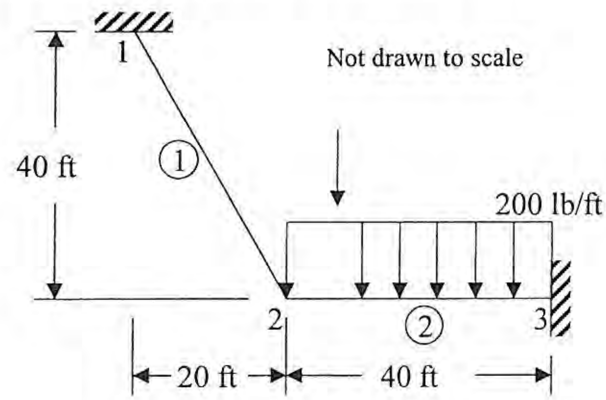
(b) [6 marks] the stress in each of the three bars.



Question 2. [20 marks] A rigid plane frame arrangement is shown in the figure. The frame is fixed at the ends, identified as nodes 1 and 3, and supports a downward acting uniformly distributed load of 200 lb/ft. Take the Young's modulus of elasticity $E = 30 \times 10^6$psi, the cross-sectional area of each element of the frame $A = 10$ in^2, and the moment of inertia $I = 200$ in^4. Determine

(a) [12 marks] the displacements and rotations at node 2, and

(b) [8 marks] the forces in each element and the reactions.



Question 3. [20 marks] A field variable $f(x,y) = x^3 y^2$ is defined over a rectangular domain $\Omega = \{\Re^+ : 0 \le x \le 8,\ 0 \le y \le 6\}$. Given the expression

$$g = \int_0^8 \int_0^6 x^3 y^2 \, dx\, dy$$

and assume the following bilinear interpolation shape functions are used to discretize the spatial/geometric variables x and y:

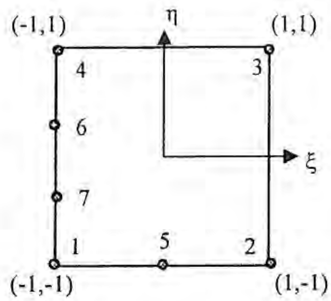
$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1+\xi)(1+\eta),\ N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

where $-1 \le \xi, \eta \le 1$ for the local coordinates ξ, η.

(a) [15 marks] Determine the value of g using the Gauss quadrature numerical integration method.

(b) [5 marks] Explain any similarity or difference between your answer and the exact solution g = 73728.

Question 4. [20 marks]

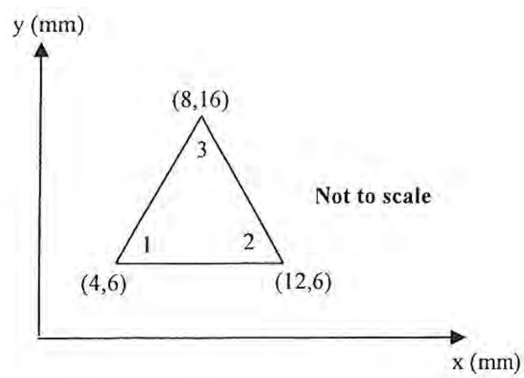


(a) [14 marks] Determine the shape functions (N_i, i = 1 to 7) of the seven-node transition element in natural/local coordinates (ξ, η) such that $-1 \leq \xi, \eta \leq 1$.
(b) [4 marks] Evaluate the shape function N_4 at the sixth node and the centroid of the element.
(c) [2 marks] Assume the field variables of the problem are displacement components denoted by u and v for the ξ and η directions, respectively. If the nodal displacement components are zero except $v_3 = v_4 = v_6 = v_7 = 0.015$ mm, determine an expression for the field variables, u and v, in the natural/ local coordinates (ξ, η).

Question 5. [20 marks] The nodal displacements for the plane strain element shown in the figure are: $u_1 = 0.002$ mm and $v_1 = 0.004$ mm; $u_2 = v_2 = 0.0$ mm; $u_3 = 0.004$ mm and $v_3 = 0.0$ mm. The plate thickness $t = 0.6$ mm, and it is made from a material with Young's modulus $E = 210$ MPa and Poisson's ratio $\nu = 0.3$. Determine

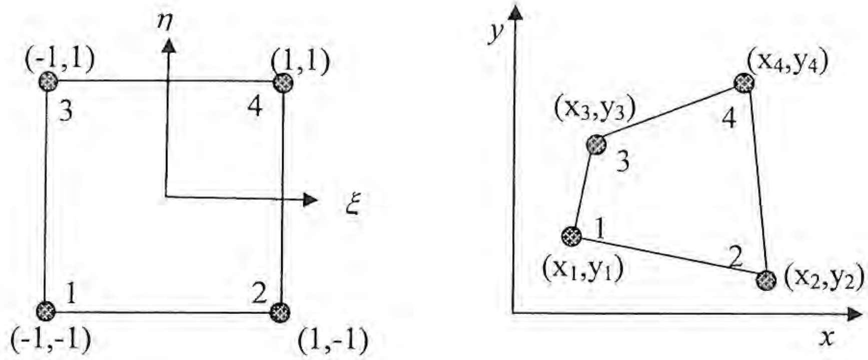
(a) [15 marks] the element stresses σ_x, σ_y, and τ_{xy}, and

(b) [5 marks] the principal stresses σ_1 and σ_2 and principal angle θ_p.



Question 6. [20 marks]

(a) [10 marks] The four-node isoparametric quadrilateral element shown below is used to map a region in the parent domain into that in the global domain.

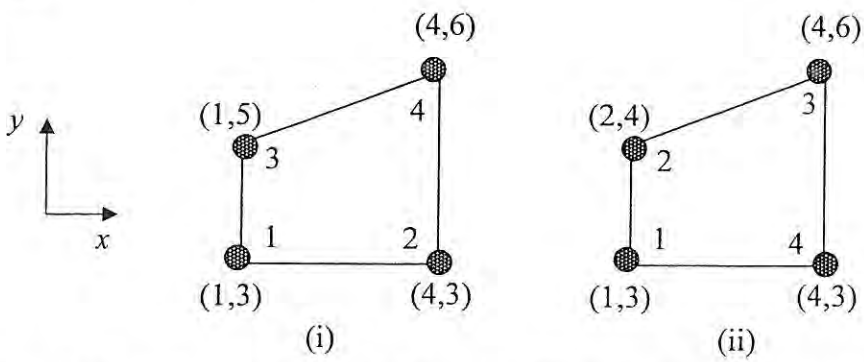


The shape functions of the element are given as

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1-\xi)(1+\eta),\ N_4 = \frac{1}{4}(1+\xi)(1+\eta)$$

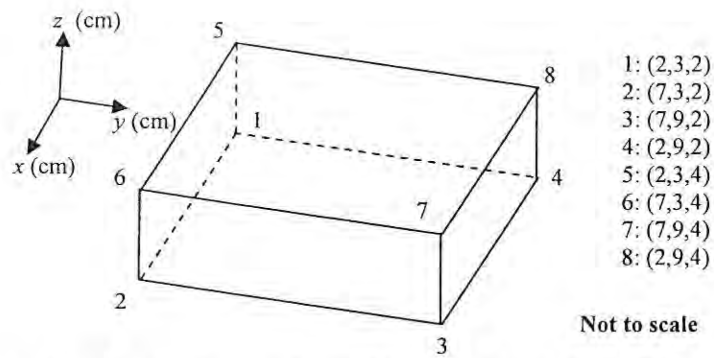
Determine the Jacobian matrix of the element.

(b) [5 marks] Use the Jacobian matrix obtained in (a) to evaluate the Jacobian of the following elements.



(c) [5 marks] What can be concluded from the Jacobian expressions obtained in (b) in light of the mapping of the coordinates?

Question 7. [20 marks]



(a) [4 marks] Determine the shape functions (N_i, $i = 1$ to 8) of an eight-node hexahedron element in natural/local coordinates (ξ, η, ζ) such that $-1 \leq \xi, \eta, \zeta \leq 1$. The node numbering is identical to that shown in the above representative global element.
(b) [3 marks] Evaluate the shape function N_8 at the second node, the eighth node, and the centroid of the element.
(c) [3 marks] Assume the field variables of the problem are displacement components denoted by $u, v,$ and w in the ξ, η, and ζ directions, respectively. If the nodal displacement components are zero except $u_2 = u_3 = u_6 = u_7 = 0.015$ mm, compute the field variables in the natural/ local coordinates (ξ, η, ζ).
(d) [7 marks] Determine the Jacobian matrix and evaluate the Jacobian of the above element.
(e) [3 marks] Determine the normal strain $\varepsilon_x = \dfrac{\partial u}{\partial x}$ and shear strain $\gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}$ at the centre of the element.

Marking Scheme

1. 20 marks total [2 parts:14 marks and 6 marks, respectively]
2. 20 marks total [2 parts: 12 marks and 8 marks, respectively]
3. 20 marks total [2 parts: 15 marks, and 5 marks, respectively]
4. 20 marks total [3 parts: 14 marks, 4 marks, and 2 marks, respectively]
5. 20 marks total [2 parts: 15 marks and 5 marks, respectively]
6. 20 marks total [3 parts: 10 marks, 5 marks, and 5 marks, respectively]
7. 20 marks total [5 parts: 4 marks, 3 marks, 3 marks, 7 marks and 3 marks, respectively]