

NATIONAL EXAMINATIONS MAY 2015

04-BS-2

PROBABILITY AND STATISTICS

2 HOURS DURATION

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.
2. “Closed Book” – no-aids other than
 - (i) A Casio or Sharp approved calculator
 - (ii) ONE hand-written information sheet (8.5”x11”), filled on both sides.
3. Any 5 questions constitute a complete paper. Only 5 questions will be marked.
4. All questions are of equal value.
5. Statistical tables of the normal, t, chi-square and F distributions are provided.
6. Questions involving hypothesis testing must be clearly formulated.

Marking Scheme

1. (a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks
2. (a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks
3. (A) (i) 5 marks (ii) 5 marks ; (B) (a) 5 marks (b) 5 marks
4. 20 marks
5. (a) 7 marks (b) 7 marks (c) 6 marks
6. (A) 10 marks ; (B) (a) 3 marks (b) 7 marks
7. (a) 10 marks (b) 10 marks
8. (a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks

1. Measurements carried out by the quality and materials control department of Super Efficient Motor Oil Company (also known as SEMOCO) revealed that the volume V of motor oil contained in the plastic bottles sold by the company is a normally distributed random variable with mean and standard deviation equal to 4,430mL and 60mL respectively.

- (a) Mr. Handyman bought a bottle of SEMOCO motor oil for the needs of his car. What is the probability that the volume V of motor oil contained in the bottle exceeded 4,400mL? Write down the probability density function of V . Then draw the probability density function of V , neatly and clearly, and indicate the area that corresponds to this probability.
- (b) Compute (i) the lower quartile and (ii) the 90th percentile of the probability distribution of V . Explain clearly the meaning of these values.
- (c) Let M represent the mean volume of motor oil contained in a random sample of four bottles. (i) Find the mean and standard deviation of the probability distribution of M . (ii) Write down the probability density function of M . (iii) Draw, neatly and clearly, the probability density function of V and M on the same diagram. (iv) Compute the probability that M is less than 4,415mL.
- (d) Mrs. Heatherspan, the owner of a fleet of trucks, bought a lot of 16 bottles of SEMOCO motor oil. Let T denote the sum of the volumes of motor oil contained in this random sample of 16 bottles. Compute the mean and variance of the probability distribution of T . Then compute the probability that T exceeds 71,000mL. Draw the probability density function of T and indicate the area that corresponds to this probability.

2. Information gathered by the Concerned Consumers Association indicates that only seventy percent of the UBL solar lights sold by a large chain of hardware stores last longer than two years.

- (a) Compute the probability that in a random sample of ten UBL solar lights more than seven will last more than two years.
- (b) Mrs. Garbo bought a box of twelve UBL solar lights. What is the probability that more than five but fewer than nine of those solar lights will last more than two years?
- (c) Use an appropriate approximation to compute the probability that in a random sample of 1,220 UBL solar lights fewer than 83 will last more than two years.
- (d) Mr. Helbling bought a lot of four UBL light bulbs to use in his flower garden. What is the probability that fewer than three will last more than two years?

3.(A)(i) On April 1st 2015 Mr. Triceps, the owner of a gymnasium, bought a lot of 10 modern treadmills from the Sporting Outlet. The next day Mr. Triceps randomly selected seven of these new treadmills and installed them in the second floor of his gymnasium. Compute the probability that at least three of these seven new treadmills will experience some problems if, unknown to Mr. Triceps, half of the treadmills he bought the Sporting Outlet need some minor adjustments from the vendor in order to function properly.

(ii) Let X denote the number of treadmills that need some adjustments from the vendor in a randomly selected sample of seven treadmills from the lot under consideration. Compute the mean and variance of the probability distribution of X.

3. (B) An extensive review of the data available in the files of the Transit Commission of a large urban centre revealed that the average number of minor delays experienced by the subway system follows the Poisson law with an average of 0.6 delays per day.

(a) Compute the probability that in a given day the subway system will experience fewer than three minor delays.

(b) What is the probability that the subway system will experience more than two but fewer than six delays in a five-day period?

4. The directors of a large professional association wish to introduce a life insurance policy for its members. Omni Canadian, the insurance company selected by the directors, offers three policies. A mail survey conducted by the association to indicate which policy the members prefer yielded the following information:

		INSURANCE POLICY			
		A	B	C	ROW TOTAL
AGE BRACKET	25-40	38	57	70	165
	41-56	68	70	62	200
	Over 56	74	33	68	175
COLUMN TOTAL		180	160	200	540

Is there any reason to conclude that the type of Insurance Policy offered is independent of the Age Bracket of the respondents? Let $\alpha = 0.05$.

5. Twenty measurements of the nitrogen content X , in pounds, of a random sample of 50lb bags of fertilizer obtained from a day's production of FETEX, a company specialized in the production of fertilizers, yielded the following information

$$\sum X = 158.00 \quad ; \quad \sum X^2 = 1,251.24$$

- (a) Use the information given above to find the 99% confidence limits of (i) the true mean and (ii) the true standard deviation of the probability distribution of X . Assume that X is a normally distributed random variable.
- (b) Test the hypothesis that the true mean of the probability distribution of X is not significantly different from 8.00lbs. Let $\alpha = 0.05$.
- (c) Test the hypothesis that the true standard deviation of the probability distribution of X is not significantly different from 0.2lbs. Let $\alpha = 0.05$.

6.(A) A random sample of 450 engineering graduates revealed that 135 plan to continue their studies while the remaining 315 plan to join the labour force. Test the hypothesis that the proportion of engineering graduates who plan to join the labour force is not significantly different from 0.75. Let $\alpha = 0.05$.

6.(B) The following table is a summary of the results obtained by the Personnel Officer of a large electric utility. The random variables X and Y denote the performance of a maintenance worker before and after a training programme. A random sample of thirteen workers was involved in the programme. (Note: The performance data were coded for convenience).

Worker i	1	2	3	4	5	6	7	8	9	10	11	12	13
X_i	16	16	15	17	12	17	16	13	11	15	16	12	14
Y_i	17	15	14	19	13	18	16	12	11	16	17	13	15

- (a) Let $d_i = Y_i - X_i$. Find the mean m_d and standard deviation s_d of the difference d_i of this sample. (Note; You may, if you wish, use the following facts:

$$\sum_{i=1}^{i=13} d_i = 6 \quad ; \quad \sum_{i=1}^{i=13} d_i^2 = 14 \quad)$$

- (b) Test the hypothesis that the training programme was successful in increasing the performance of the workers against the alternative that it did not affect the performance. Formulate the null and alternative hypotheses clearly and neatly and explain the implications of the conclusion you reached. Let $\alpha = 0.05$. Assume that the difference under consideration is a normally distributed random variable.

7. The following data pertain to the rating, in lumens/watt, of two makes of equally priced LED light bulbs. Originally twelve bulbs of each make were randomly selected for that purpose. However because of some unfortunate clerical mistake three results had to be rejected. The remaining results are tabulated below.

	Make A	Make B
Sample size	$n_A = 11$	$n_B = 10$
Sample mean	$m_A = 84$	$m_B = 76$
Sample standard deviation	$s_A = 0.25$	$s_B = 0.35$

(a) Test the hypothesis that the variability of the rating of the LED light bulb of Make A is not significantly different from that of Make B. Let $\alpha = 0.05$. State, clearly and neatly, any assumptions you need to make.

(b) Test the hypothesis that the mean rating of the LED light bulb of Make A is not significantly different from that of Make B. Let $\alpha = 0.05$.

8. The following data represent the number of years of service Y and the number of days of absence X during the past year for sixteen randomly selected employees working for a large hardware store.

$$\sum_{i=1}^n X_i = 90 \quad ; \quad \sum_{i=1}^n X_i^2 = 600 \quad ; \quad \sum_{i=1}^n Y_i = 56;$$

$$\sum_{i=1}^n Y_i^2 = 296 \quad ; \quad \sum_{i=1}^n X_i Y_i = 231 \quad ; \quad n = 16$$

(a) Compute the coefficient of correlation r of X and Y .

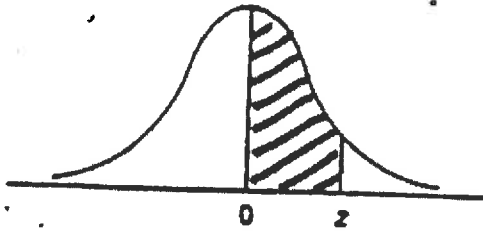
(b) Find the 95% confidence limits of the true coefficient of correlation ρ .

(c) It is believed that Y and X are related by an equation of the form $Y = \beta_0 + \beta_1 X + \epsilon$.

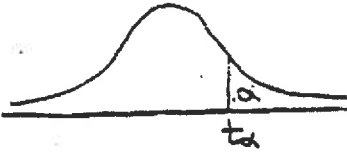
Write down the normal equations of the least squares line and then compute the estimates b_0 and b_1 of β_0 and β_1 respectively.

(d) Find the 95% confidence limits of the slope β_1 .

NORMAL DISTRIBUTION TABLE



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990



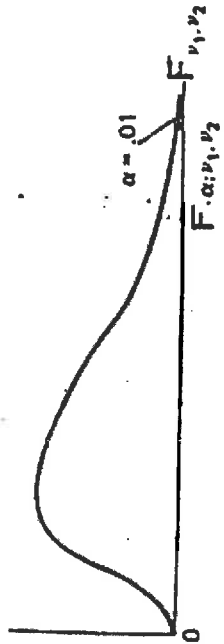
t- Distribution

d.f	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	d.f
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
inf.	1.282	1.645	1.960	2.326	2.576	inf.

THE CHI-SQUARE DISTRIBUTION

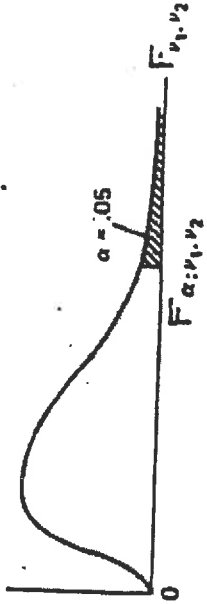


df	Probability that chi-square value will be exceeded							
	.995	.990	.975	.950	.050	.025	.010	.005
1	---	---	---	.004	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	41.34	44.46	48.23	50.99
29	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	79.08	83.30	88.33	91.95
70	43.28	45.44	48.76	51.74	90.53	95.02	100.43	104.22
80	51.17	53.54	57.15	60.39	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	124.34	129.56	135.81	140.17



Upper 1% points

ν_1	ν_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞	
1	4052																				
2	4999.5	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00
3	34.12	30.82	29.46	28.24	27.12	26.07	25.07	24.11	23.19	22.31	21.46	20.64	19.85	19.08	18.34	17.62	16.92	16.24	15.58	14.94	14.32
4	21.20	18.00	16.69	15.52	14.98	14.37	13.79	13.24	12.71	12.20	11.70	11.22	10.75	10.30	9.86	9.44	9.03	8.63	8.24	7.86	7.50
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.26	10.07	9.89	9.72	9.56	9.41	9.26	9.11	8.97	8.83	8.69	8.56	8.43	8.31
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.76	7.66	7.56	7.46	7.37	7.28	7.19	7.10	7.01	6.92	6.83
7	12.25	9.53	8.43	7.83	7.46	7.19	6.99	6.84	6.72	6.62	6.52	6.42	6.32	6.23	6.14	6.05	5.96	5.87	5.78	5.69	5.60
8	11.26	8.63	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.72	5.62	5.52	5.43	5.34	5.25	5.16	5.07	4.98	4.89	4.80
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.16	5.06	4.96	4.87	4.78	4.69	4.60	4.51	4.42	4.33	4.24
10	10.04	7.56	6.53	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.75	4.65	4.56	4.47	4.38	4.29	4.20	4.11	4.02	3.93	3.84
11	9.63	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.44	4.34	4.25	4.16	4.07	3.98	3.89	3.80	3.71	3.62	3.53
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.20	4.10	4.01	3.92	3.83	3.74	3.65	3.56	3.47	3.38	3.29
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.00	3.90	3.81	3.72	3.63	3.54	3.45	3.36	3.27	3.18	3.09
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.84	3.74	3.65	3.56	3.47	3.38	3.29	3.20	3.11	3.02	2.93
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.70	3.60	3.51	3.42	3.33	3.24	3.15	3.06	2.97	2.88	2.79
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.59	3.49	3.40	3.31	3.22	3.13	3.04	2.95	2.86	2.77	2.68
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.49	3.39	3.30	3.21	3.12	3.03	2.94	2.85	2.76	2.67	2.58
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.41	3.31	3.22	3.13	3.04	2.95	2.86	2.77	2.68	2.59	2.50
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.33	3.23	3.14	3.05	2.96	2.87	2.78	2.69	2.60	2.51	2.42
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.27	3.17	3.08	2.99	2.90	2.81	2.72	2.63	2.54	2.45	2.36
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.21	3.11	3.02	2.93	2.84	2.75	2.66	2.57	2.48	2.39	2.30
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.16	3.06	2.97	2.88	2.79	2.70	2.61	2.52	2.43	2.34	2.25
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.11	3.01	2.92	2.83	2.74	2.65	2.56	2.47	2.38	2.29	2.20
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.07	2.97	2.88	2.79	2.70	2.61	2.52	2.43	2.34	2.25	2.16
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	3.03	2.93	2.84	2.75	2.66	2.57	2.48	2.39	2.30	2.21	2.12
26	7.72	5.53	4.64	4.14	3.82	3.60	3.42	3.29	3.18	3.09	2.99	2.89	2.80	2.71	2.62	2.53	2.44	2.35	2.26	2.17	2.08
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.96	2.86	2.77	2.68	2.59	2.50	2.41	2.32	2.23	2.14	2.05
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.93	2.83	2.74	2.65	2.56	2.47	2.38	2.29	2.20	2.11	2.02
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.90	2.80	2.71	2.62	2.53	2.44	2.35	2.26	2.17	2.08	1.99
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.88	2.78	2.69	2.60	2.51	2.42	2.33	2.24	2.15	2.06	1.97
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.70	2.60	2.51	2.42	2.33	2.24	2.15	2.06	1.97	1.88	1.79
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.53	2.43	2.34	2.25	2.16	2.07	1.98	1.89	1.80	1.71	1.62
120	6.83	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.37	2.27	2.18	2.09	2.00	1.91	1.82	1.73	1.64	1.55	1.46
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.22	2.12	2.03	1.94	1.85	1.76	1.67	1.58	1.49	1.40	1.31



Upper 5% points

v_1	v_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.2	251.1	252.2	253.3	254.3
2	1	18.51	19.00	19.16	19.23	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	1	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	1	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	1	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	1	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.66
7	1	5.39	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	1	5.12	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	1	4.96	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	1	4.84	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	1	4.75	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	1	4.67	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	1	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.38	2.42	2.38	2.34	2.30	2.25	2.21
14	1	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.32	2.35	2.31	2.27	2.22	2.18	2.13
15	1	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.29	2.25	2.20	2.16	2.11	2.07
16	1	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.24	2.19	2.15	2.10	2.06	2.01
17	1	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.19	2.15	2.10	2.06	2.01	1.96
18	1	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
19	1	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
20	1	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
21	1	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
22	1	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.75
23	1	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
24	1	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
25	1	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
26	1	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.83	1.78	1.73	1.67
27	1	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
28	1	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
29	1	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
30	1	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
40	1	3.92	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
60	1	3.84	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
120	1	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00



Upper 2.5% points

v_1	v_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
1	2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
1	3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
1	4	12.22	10.63	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
1	5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
1	6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
1	7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
1	8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
1	9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
1	10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
1	11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
1	12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.27	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
1	13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
1	14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
1	15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
1	16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
1	17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
1	18	5.98	4.56	3.95	3.60	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
1	19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
1	20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
1	21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
1	22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
1	23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
1	24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
1	25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
1	26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
1	27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
1	28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
1	29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
1	30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
1	40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
1	60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
1	120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
1	∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00