# National Exams May 2015

**07-Elec-A2, Systems & Control**

3 hours duration

**NOTES:**

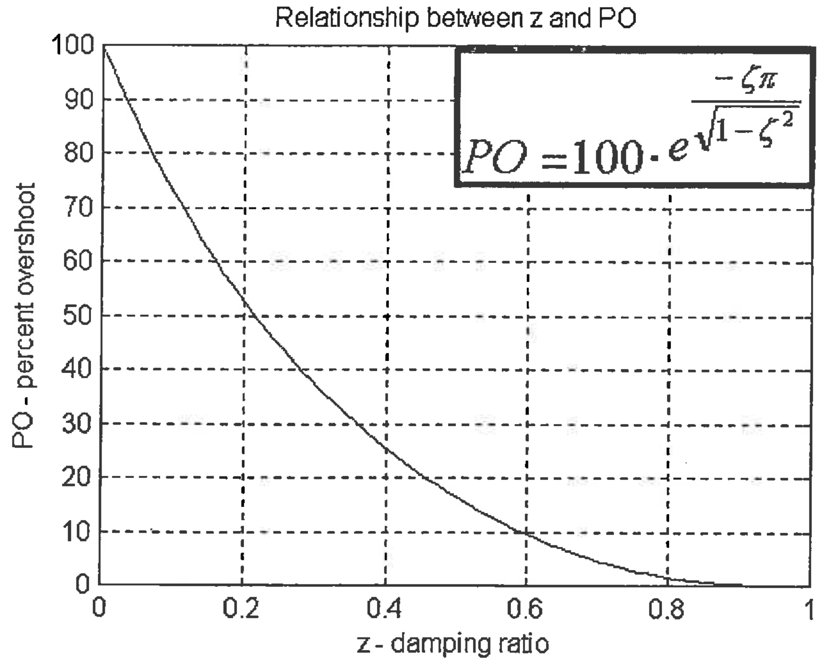
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet has to be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate answers to which questions should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

| **YOUR MARKS** | | |
|---|---|---|
| **QUESTIONS 1 AND 2 ARE COMPULSORY:** | | |
| Question 1 | 20 | |
| Question 2 | 20 | |
| **CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:** | | |
| Question 3 | 20 | |
| Question 4 | 20 | |
| Question 5 | 20 | |
| Question 6 | 20 | |
| Question 7 | 20 | |
| Question 8 | 20 | |
| **TOTAL:** | 100 | |

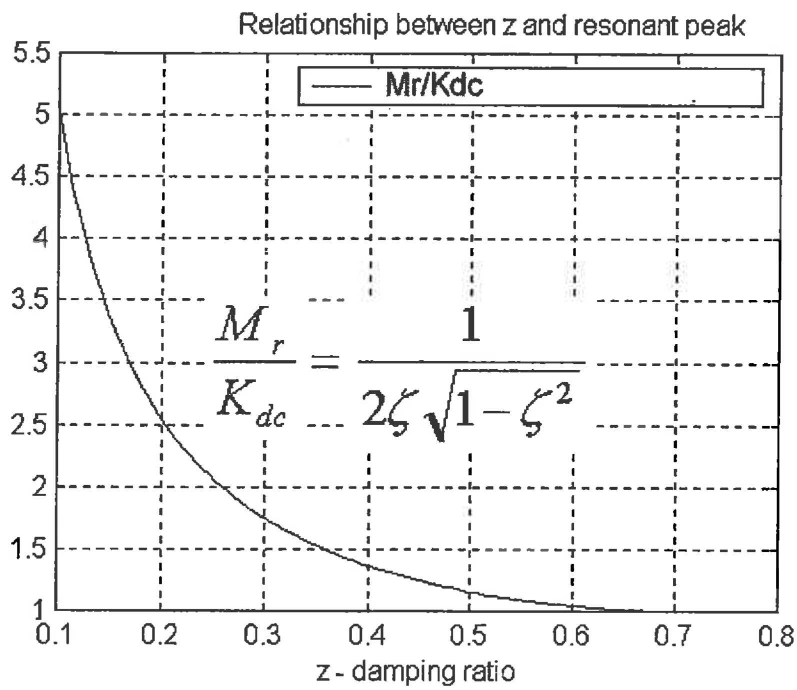
# A Short Table of Laplace Transforms

| Laplace Transform | Time Function |
|---|---|
| $1$ | $\sigma(t)$ |
| $\frac{1}{s}$ | $1(t)$ |
| $\frac{1}{(s)^2}$ | $t \cdot 1(t)$ |
| $\frac{1}{(s)^{k+1}}$ | $\frac{t^k}{k!} \cdot 1(t)$ |
| $\frac{a}{s+a}$ | $e^{-at} \cdot 1(t)$ |
| $\frac{a}{(s+a)^2}$ | $te^{-at} \cdot 1(t)$ |
| $\frac{a}{s(s+a)}$ | $(1-e^{-at}) \cdot 1(t)$ |
| $\frac{a}{s^2+a^2}$ | $\sin at \cdot 1(t)$ |
| $\frac{s}{s^2+a^2}$ | $\cos at \cdot 1(t)$ |
| $\frac{s+a}{(s+a)^2+b^2}$ | $e^{-at} \cdot \cos bt \cdot 1(t)$ |
| $\frac{b}{(s+a)^2+b^2}$ | $e^{-at} \cdot \sin bt \cdot 1(t)$ |
| $\frac{a^2+b^2}{s[(s+a)^2+b^2]}$ | $\left(1-e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$ |
| $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ | $\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$ |
| $\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$ | $\left(1-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t+\cos^{-1}\zeta\right)\right) \cdot 1(t)$ |
| $F(s) \cdot e^{-Ts}$ | $f(t-T) \cdot 1(t)$ |
| $F(s+a)$ | $f(t) \cdot e^{-at} \cdot 1(t)$ |
| $sF(s)-f(0+)$ | $\frac{df(t)}{dt}$ |
| $\frac{1}{s}F(s)$ | $\int_{0+}^{+\infty} f(t)dt$ |

# Useful Plots




PO vs. Damping Ratio




Resonant Peak vs. Damping Ratio

## Question 1 (Compulsory)

*PID Controller Design, Stability, Steady State Error, Transient Specifications, Dominant Poles Model.*

Consider a closed loop control system working in a feedback configuration under Proportional + Rate Feedback Control, shown in Figure Q1.1 below.

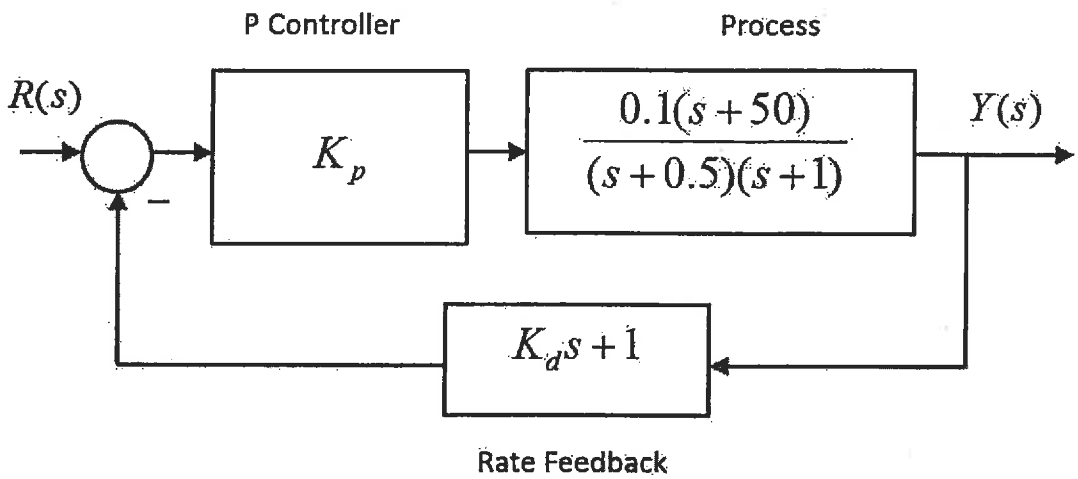



**Figure Q1.1: Closed Loop System under P + Rate feedback Control**

1) **(8 marks)** Derive an expression for the closed loop system transfer function in terms of the two Controller Gains, the Proportional Gain, $K_p$, and the Rate Feedback Gain, $K_d$.
2) **(5 marks)** Determine the value of the Proportional Gain, $K_p$, such that the closed loop step response will have a Steady State Error of 5%. Substitute this value into the expression for the closed loop system transfer function, so that only one unknown, the Rate Feedback Gain, $K_d$, is left.

3) **(7 points)** Next, determine an APPROXIMATE value of the Rate Feedback Gain, $K_d$, such that the closed loop step response will have a Percent Overshoot of 15%. To do so, make a simplifying assumption about the closed loop transfer function derived in Item 1.

   HINT: There are two ways such a simplifying assumption can be made. Both will result in a considerable reduction of work involved. Specify very clearly the assumption you make.

## Question 2 (Compulsory)

*Root Locus Analysis and Gain Selection, Transient Response Specifications, Second Order Model.*

Consider a closed loop control system working in a unit feedback configuration under Proportional Control, where the process transfer function is described as follows:

$$G(s) = \frac{10}{(s+2)^2(s+10)}$$

**PART A (10 marks)** - *Root Locus Analysis*

Sketch a Root Locus plot for this system and place it in Figure Q2.1 - show all relevant coordinates, such as the crossovers through the Imaginary Axis, the break-away, the centroid and the asymptotic angles.

**PART B (10 marks)** - *Root Locus Gain Selection*

Next, determine the value of the Operational Gain ($K_{op}$) such that the closed loop step response will have a Percent Overshoot of 5%.

For the computed value of the Operational Gain, $K_{op}$, answer these questions:

a. What will be the steady state error, in %, of the closed loop step response?
b. What will be the Settling Time, $T_{settle(\pm 2\%)}$ of the closed loop step response?
c. What will be the system Gain Margin?

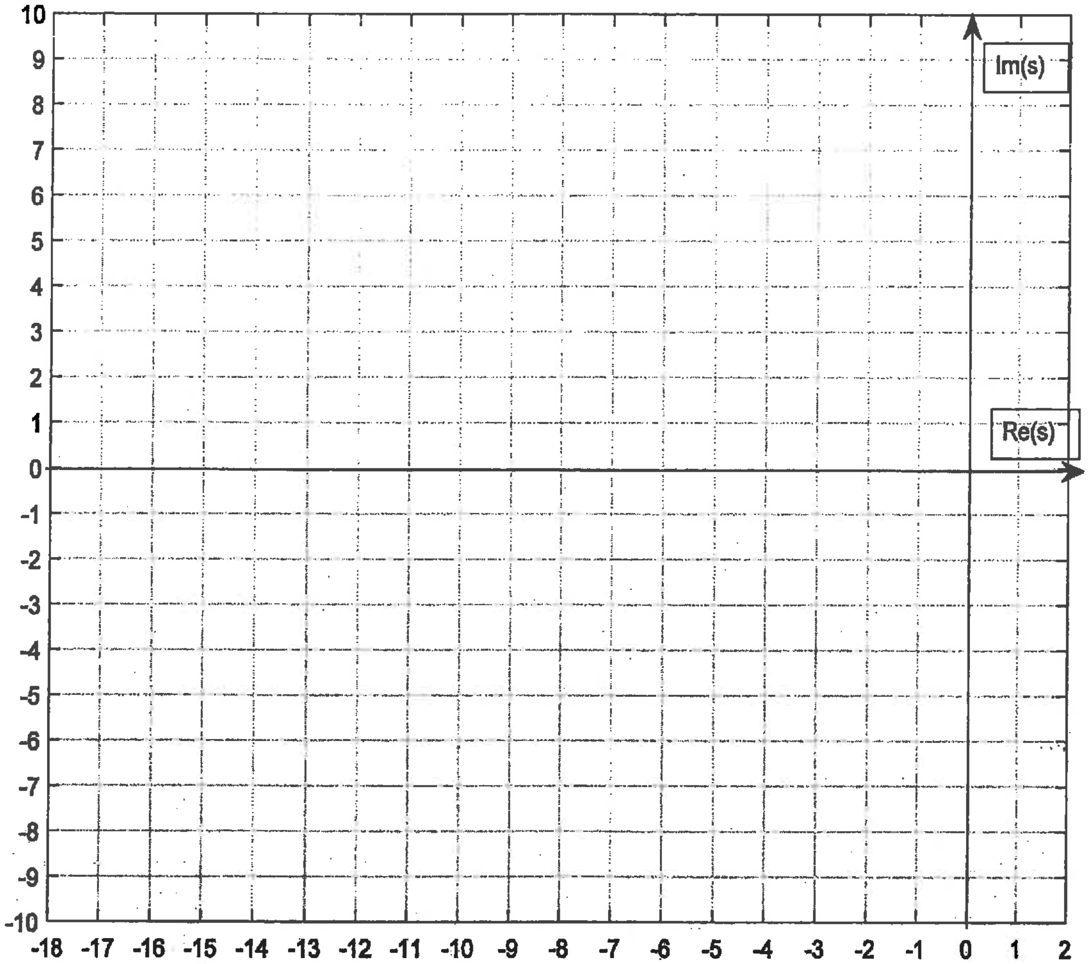



**Figure Q2.1** – Place Root Locus Plot here

## Question 3

*"Top-Down" Controller Design through Pole Matching, Second Order Dominant Poles Model, Effect of Additional Poles and Zeros on the Closed Loop Response, Step Response Specifications.*

Consider a unit feedback control system where the process transfer function is described below as G(s), and the Controller transfer function is described below as $G_c(s)$:

$$G(s) = \frac{10}{s(s+2)}$$

$$G_c(s) = \frac{a_1 s + a_0}{b_1 s + 1}$$

1. **(7 marks)** Derive the closed loop transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$, in terms of the controller parameters, $a_1, a_0$ and $b_1$, and write the system Characteristic Equation, $Q(s) = 0$.
2. **(5 marks)** The compensated closed loop step response of this system is to have the following specifications: Percent Overshoot, $PO = 5\%$, Settling Time, $T_{settle(\pm 2\%)} = 4$ sec, and Steady State Error, $e_{ss(\%)} = 0\%$. Identify the "Desired' closed loop Characteristic Equation that would meet the specifications.

   HINT: Assume that the closed loop system has two dominant poles with a damping ratio $\zeta$ and the natural frequency $\omega_n$ corresponding to the desired step response specifications, and that the third closed-loop pole is at a location **ten (10) times** further to the left in the s-plane than the Real Part of the dominant complex poles.
3. **(8 marks)** Next, find appropriate controller parameters, $a_1, a_0$ and $b_1$, by matching the system Characteristic Equation with the "Desired" Characteristic Equation, and identify the nature of the controller i.e. is it a PD, PI, PID, Lead or Lag Controller.

## Question 4

*System Stability in the s-domain and in the frequency domain: Bode plots, Root Locus plots and Routh-Hurwitz Criterion of Stability.*

A unit feedback control system that is unstable in an open loop configuration, is to be stabilized using a Proportional + Integral + Derivative (PID) Controller. The combined process-controller Open Loop transfer function is described as follows:

$$G_{open}(s) = K_p\left(1 + \frac{1}{T_i s}\right)\cdot(1 + T_d s)\cdot\frac{10}{s(s+20)(s-1)}$$

The PID controller time constants are as follows: $\boldsymbol{T_i = 2}$ seconds and $\boldsymbol{T_d = 0.5}$ seconds. A frequency response (Bode plot) for the compensated open loop system is shown in Figure Q4.1, and the corresponding Root Locus plot is shown in Figure Q4.2.

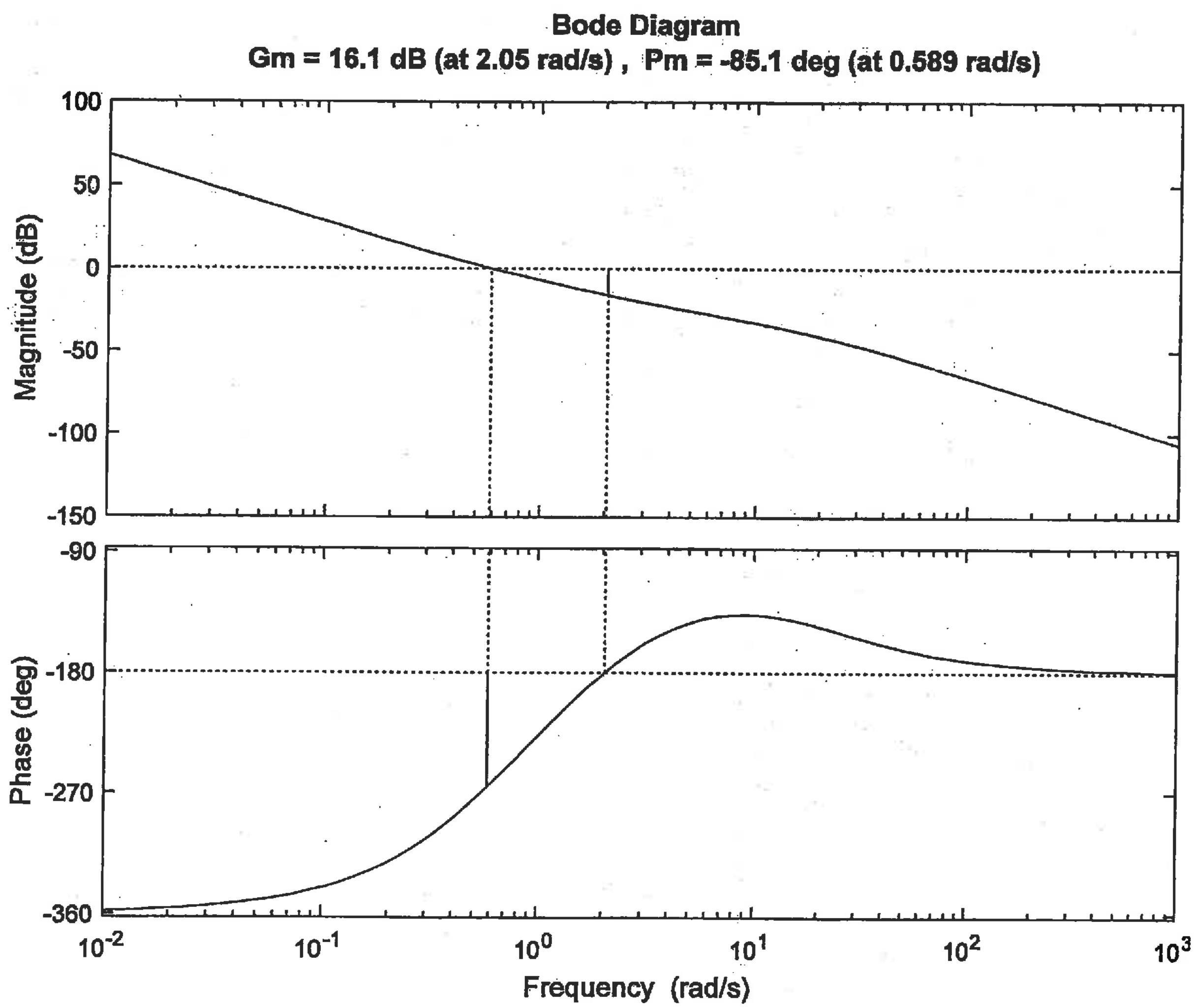



**Figure Q4.1** – Open Loop Compensated System Frequency Response

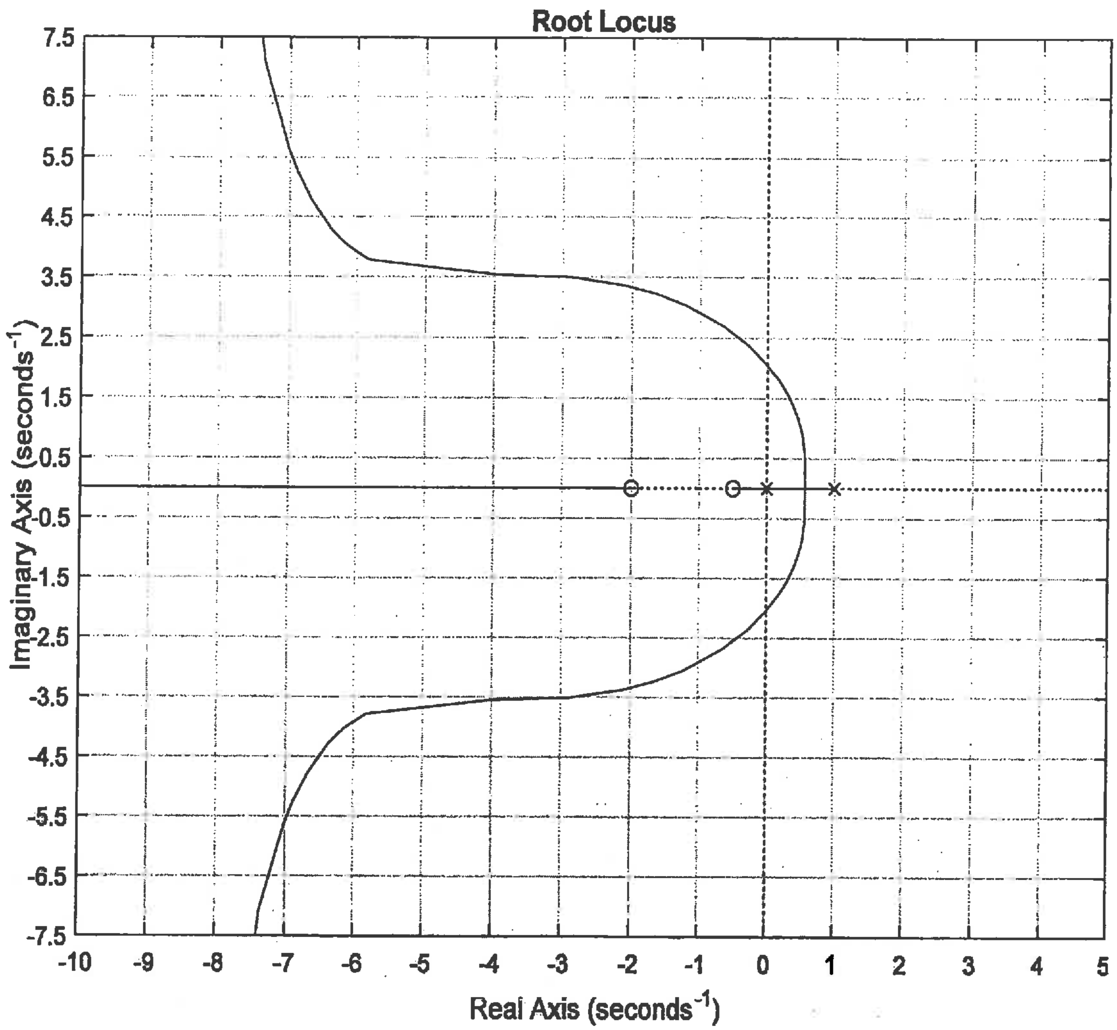



**Figure Q4.2** – Root Locus of the PID-Compensated System

1. **(10 marks)** Use the information provided in Figure Q4.1 & Figure Q4.2 to determine the range of Proportional Gain, $K_p$, for a safe, stable operation of the closed loop system.
2. **(10 marks)** Verify the range of Proportional Gain, $K_p$, found in item 1) for the safe, stable operation of the closed loop system, by using the Routh-Hurwitz Criterion of Stability.

## Question 5

*Second Order Dominant Poles Model from Frequency Response Plots, Step Response Specifications and Steady State Errors.*

Consider a closed loop unit feedback control system with a process transfer function as follows:

$$G(s) = \frac{20(s+100)}{(s+2)(s+5)(s+20)}$$

**PART A (5 marks)** – *Closed Loop Model from Closed Loop Frequency Response.*

Figure Q5.1 shows the uncompensated closed loop frequency response plot (magnitude only). Based on the information provided, create an equivalent second order model for the uncompensated closed loop system. Calculate the model parameters: DC Gain, $K_{dc}$, the damping ratio, $\zeta$, and the frequency of natural oscillations, $\omega_n$, and write the model transfer function, $G_{m1}(s)$.

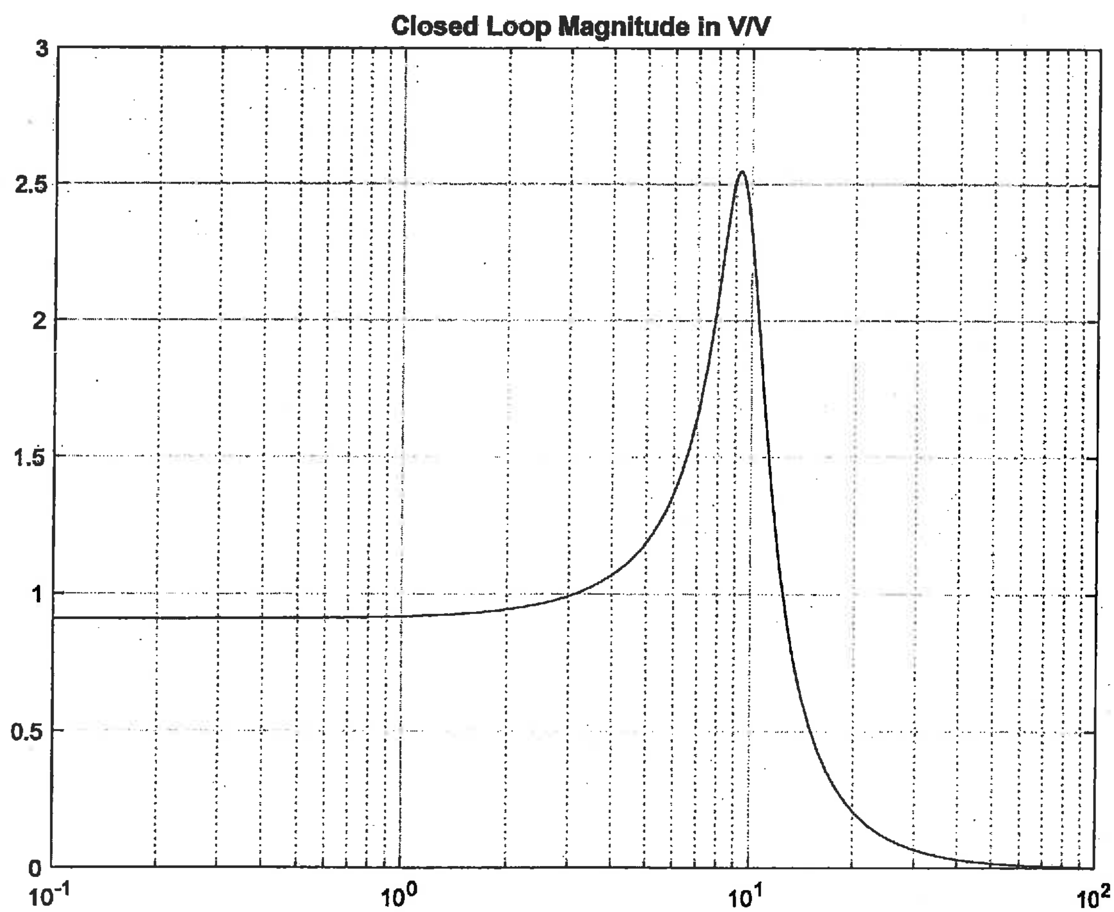



**Figure Q5.1** – Closed Loop Uncompensated System

**PART B (5 marks)** – *Closed Loop Model from Open Loop Frequency Response.*

Figure Q5.2 shows the uncompensated open loop frequency response plots (both magnitude and phase). Based on the information provided, create another equivalent second order model for the uncompensated closed loop system. Calculate the model parameters: DC Gain, $K_{dc}$, the damping ratio, $\zeta$, and the frequency of natural oscillations, $\omega_n$, and write the model transfer function, $G_{m2}(s)$.

**PART C (5 marks)** – *Closed Loop Model from Dominant Poles.*

The uncompensated closed loop transfer function of the system is calculated as follows:

$$G_{cl}(s) = \frac{20(s + 100)}{(s + 23.74)(s^2 + 3.26s + 92.66)}$$

Based on the information provided, create yet another equivalent second order model for the uncompensated closed loop system. Calculate the model parameters: DC Gain, $K_{dc}$, the damping ratio, $\zeta$, and the frequency of natural oscillations, $\omega_n$, and write the model transfer function, $G_{m3}(s)$.

**PART D (5 marks)** – *Closed Loop Step Response Estimates.*

Are the three models consistent? Use either one to estimate the following time response specifications of the uncompensated closed loop step response: Percent Overshoot, Settling Time and Steady State Error (PO, $T_{settle(\pm 2\%)}$ and $e_{ss\%}$, respectively).

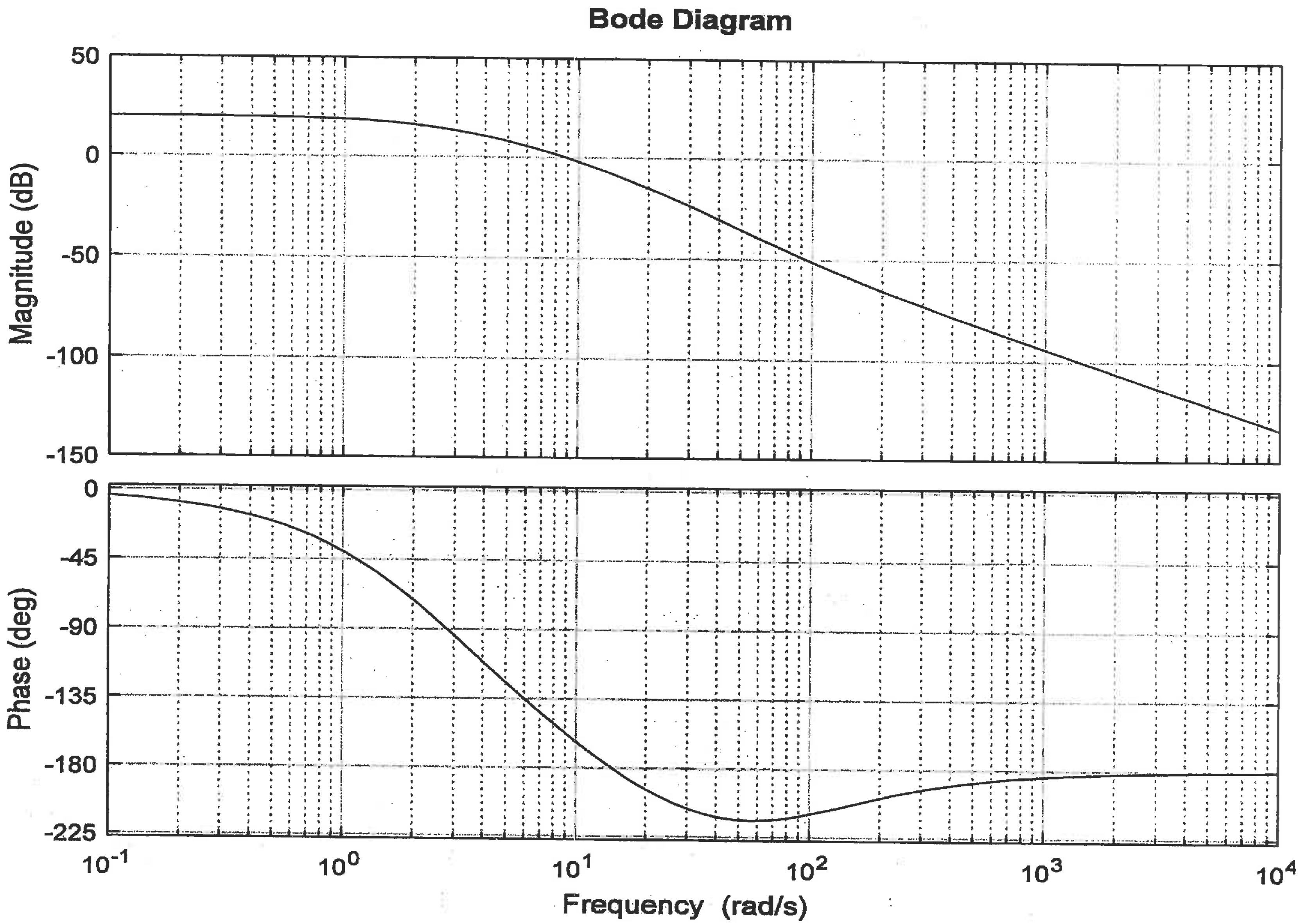



**Figure Q5.2** – Open Loop Uncompensated System Frequency Response

## Question 6

*Controller design in Frequency Domain – Lag Controller, Step Response Specifications.*

Consider the same closed loop unit feedback control system as in Question 5, with the uncompensated open loop frequency response plots shown in Figure Q5.2, on page 12. The closed loop system is to be compensated by a Lag Controller with the transfer function described below as $G_c(s)$. Note that for $G_c(s)$ to represent the Lag-type Controller transfer function, the pole at $-\frac{1}{b_1}$ has to be closer to the Imaginary axis than the zero at $-\frac{a_0}{a_1}$.

$$G_c(s) = \frac{a_1 s + a_0}{b_1 s + 1}$$

The design requirements are:

- The Steady State Error for the unit step input for the compensated closed loop system is equal to 2%.
- The compensated Phase Margin is to be 45 degrees ($\Phi_m = 45^0$) and the corresponding Crossover Frequency for Phase Margin is to be 4 rad/sec ($\omega_{cp} = 4$).

1. **(15 marks)** Calculate the appropriate Lag Controller parameters and the Controller transfer function. Show the general shape of the compensated frequency response by overlaying it on top of the uncompensated plot in Figure Q5.2.

2. **(5 marks)** Estimate the following time response specifications of the compensated closed loop step response: Percent Overshoot, Settling Time and Steady State Error (PO, $T_{settle(\pm 2\%)}$ and $e_{ss\%}$, respectively).

## Question 7

*State Space Model, System Eigenvalues, Stability, Pole Placement by State Feedback Method, Steady State Errors to Step Inputs.*

Consider the following state space representation of a certain process G(s):

$$\dot{x_1} = x_1 + x_2 + u$$
$$\dot{x_2} = x_1 + x_2$$
$$y = 2x_1 - x_2$$

1. **(8 marks)** Determine if the system is controllable and observable.

2. **(7 marks)** A control system is to be built around the process in a unity feedback configuration with a reference input *r* by utilizing a state-variable feedback according to the following equation:

$$u = r - K_1 \cdot x_1 - K_2 \cdot x_2$$

Determine the values of the gains in the state feedback vector **k** so that the resulting closed loop system will have eigenvalues at: -2 and -3 and the steady-state error to a step input signal *r* is to be zero.

3. **(5 marks)** Find the closed loop system transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$ for the values of $K_1$ and $K_2$ found above.

## Question 8

**PART A (12 marks)** – *State Space Model vs. Transfer Functions, Mason's Gain Formula.*

Consider a linear process described by the signal flow graph in Figure Q8.1.

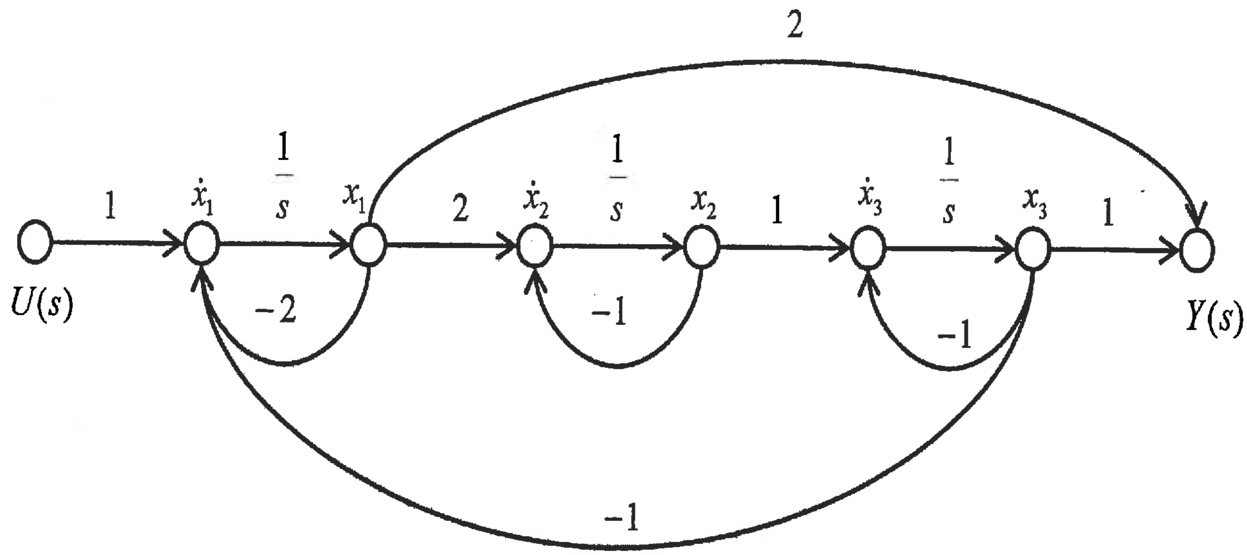



**Figure Q8.1**

Derive a set of the corresponding state equations - follow the choice of state variables as indicated in Figure Q8.1. Next, determine the process transfer function $G(s) = \frac{Y(s)}{U(s)}$.

**PART B (8 marks)** – *Nyquist Criterion of Stability.*

A certain open loop control system has **three poles** on the left-hand side of the s-plane (i.e. in LHP), and **one pole** on the right-hand side (i.e. in RHP). The corresponding Nyquist diagram was generated using a clockwise (CW) $\Gamma$ path in the s-plane, and is shown in Figure Q8.2.

1) Apply the Nyquist criterion of stability to determine if the system is stable;
2) How many closed loop poles are in LHP, and how many, if any, are in the RHP?
3) Calculate the system Gain Margin, as a V/V ratio, as well as in decibels.

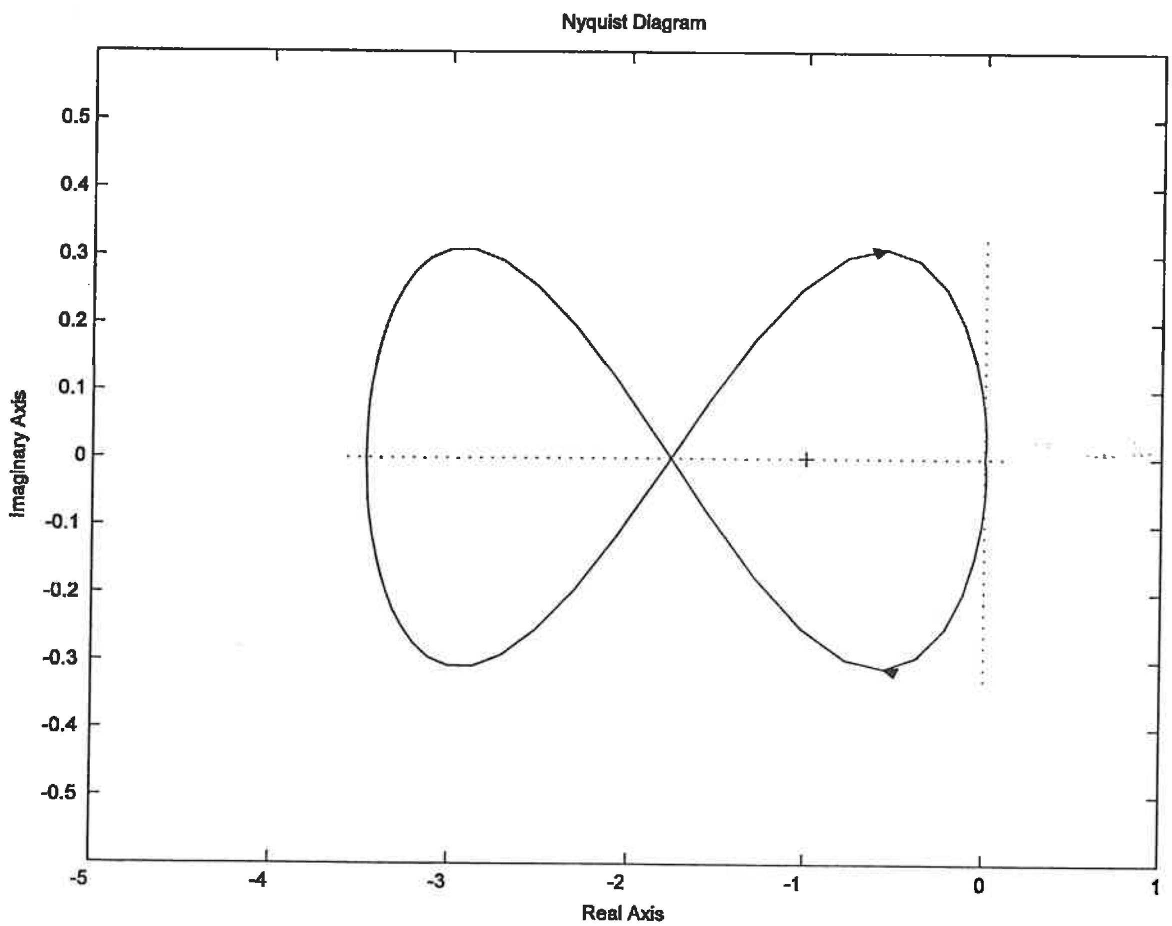



**Figure Q8.2**