NATIONAL EXAMS May 2015
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio . or a Sharp

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

5. All questions are of equal value.

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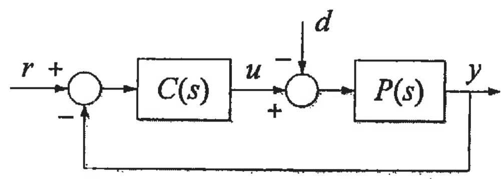
1. Consider the vehicle cruise control system below with, $P(s)=\dfrac{100}{10s+1}$, $C(s)=\dfrac{10}{5s+1}$

(a) The vehicle is moving with constant steady state speed along a level road when suddenly the grade changes to a fixed incline corresponding to a unit step disturbance torque, *d*. Determine the steady state error in speed, *r* - *y*.



(b) The vehicle encounters an undulating road resulting in a disturbance torque of $d(t)=3\sin(0.5t)$. Determine the steady state error in speed.

(c) Determine the phase margin.

(d) Explain one way to alter *C*(*s*) to improve the phase margin and not compromise the steady state tracking error.

2. Consider the dynamic system with input, *u*(*t*), and the output, $y(t)$.

$$\dot{\theta}(t)=-2\theta(t)-\gamma(t)+u(t)$$
$$\dot{\gamma}(t)=\theta(t)$$
$$\dot{h}(t)=\gamma(t)$$
$$y(t)=\gamma(t)+h(t)$$

(a) Determine a state space model for the system.

(b) Determine the response *y*(*t*) when *u*(*t*) = 0, $\theta(0)=1$, $\gamma(0)=0$ and $h(0)=0$.

(c) Determine the transfer function relating *Y*(*s*) to *U*(*s*).

(d) Justify whether the system is *bounded-input-bounded-output* stable?

(e) Justify whether the systems is (i) completely controllable, (ii) completely observable?

3. Input and output measurements from a system are to be used to fit a discrete model of the form, $Y(z)=P(z)U(z)$, where, $P(z)=\dfrac{\beta}{z-\alpha}$. It is known that the measurements are contaminated by zero mean white noise.

(a) Measurements of *u*(*k*) and *y*(*k*) are taken at time instants, *k*, as listed in the Table below. Find a least squares estimate for α and β.

k	0	1	2	3	4	5	6
y(*k*)	0	10	4	3	1.6	0.4	0.3
u(*k*)	1	0	0	0	0	0	0

(b) If *u*(*k*) = 2, what is the steady state output as predicted by the identified model?

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4. Consider the system,

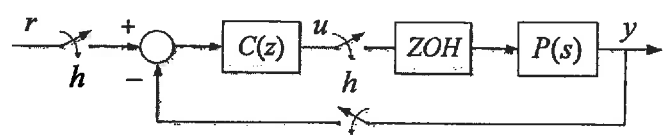
$$\dot{x}(t) = \begin{pmatrix} -1 & -2 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix} x(t) + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} x(t)$$

Design a statefeedback controller of the form $u(t) = Lr(t) - Kx(t)$, i.e., determine L and K such that the closed loop poles are $s = -10$, $s = -3 + j4$, $s = -3 - j4$, and the steady state tracking error, $e = r - y$, is zero when $r(t)$ is a step input.

5. Consider the sampled data and digital control system below. The input to the ZOH and the (continuous) output, y, are uniformly sampled with a sample period of $h = 0.2$ s. $C(z)$ and $P(s)$ are given by,

$$C(z) = \frac{K}{z-1}, \quad P(s) = \frac{1}{s+1}$$

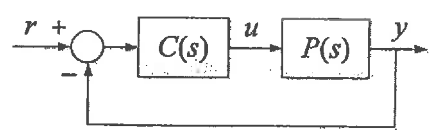


(a) Determine the discrete closed loop transfer function, $T(z)$, that relates $Y(z)$ to $R(z)$.

(b) Determine the range of values of K for stability.

(c) Assuming stability, determine the steady state tracking error for a unit ramp input. Comment on the inter-sample behavior at $y(t)$.

6. Consider the feedback system below with, $C(s) = K$, $P(s) = e^{-s}$.



(a) Determine the range of K such that the gain margin is at least 6 dB. Determine the corresponding phase margin.

(b) Assuming stability, determine the steady state tracking error, $e(t) = r(t) - y(t)$, as a function of K.

(c) Determine the unit step response for $K = 1.0$.

(d) Redesign $C(s)$ such that: i) the steady state tracking error is zero for a step input and ii) the gain margin is at least 6 dB.

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Inverse Laplace Transforms	
$F(s)$	**$f(t)$**
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^{n}e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\dfrac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\dfrac{2t^{n}e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	**$f(nT)$**
$\dfrac{Kz}{z-a}$	Ka^{n}
$\dfrac{\left(C+jD\right)z}{z-re^{j\varphi}}+\dfrac{\left(C-jD\right)z}{z-re^{-j\varphi}}$	$2r^{n}\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{\left(z-a\right)^{r}}$, $r=2,3\ldots$	$\dfrac{Kn\left(n-1\right)\ldots\left(n-r+2\right)}{\left(r-1\right)!a^{r-1}}a^{n}$

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Table of Laplace and z-Transforms (*h* denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos \beta h)}{z^2-2z\cos \beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin \beta h}{z^2-2z\cos \beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos \beta h)}{z^2-2ze^{-\alpha h}\cos \beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin \beta h}{z^2-2ze^{-\alpha h}\cos \beta h+e^{-2\alpha h}}$
$t f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t} f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$