

National Exams May 2015

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio or Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.
3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Question 1:

Construct asymptotic Bode magnitude plots for the following transfer functions.

(a)
$$\frac{4}{s + 2}$$

(b)
$$\frac{4}{(0.4s + 1)(s + 1)}$$

Question 2:

(a) Calculate the unit step response of

$$G(s) = \frac{1}{(s + 2)^2 (s + 1)}$$

(b) Calculate the unit step response of the system

$$G(s) = \frac{54}{(2s + 6)(s^2 + 3s + 9)}$$

Question 3:

Let Fig. 1 model a temperature control system with plant transfer function $G(s) = 1/[(s + 1)(s + 5)]$.

- With P control $G_c = K_c$, what is the system type number, and what is the gain?
- For $G_c = K_c$, find K_c for a damping ratio 0.5 and the corresponding steady-state error for a unit step input.

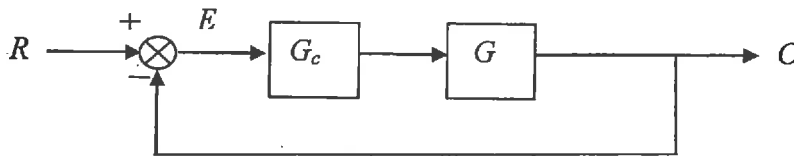


Figure 1

Question 4:

In the system with rate feedback shown in Fig. 2:

- Sketch the root locus and find K for a system damping ratio 0.5 for the dominating poles.
- Find the steady-state errors for step and ramp inputs for K of part (a).

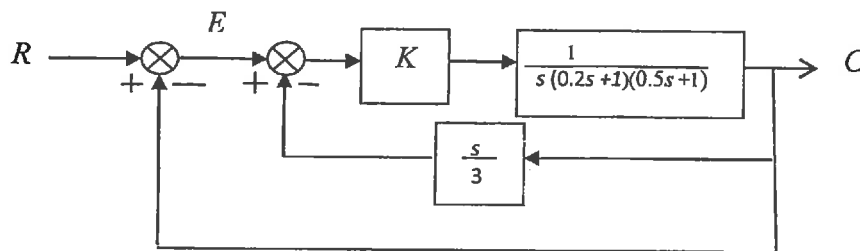


Figure 2

Question 5:

Use the Routh-Hurwitz stability criterion to determine the stability of systems with the following characteristic equations.

(a) $s^4 + 10s^3 + 33s^2 + 46s + 30 = 0$

(b) $s^4 + s^3 + 3s^2 + 2s + 5 = 0$

(c) $s^3 + 2s^2 + 3s + 6 = 0$

Question 6:

In Fig. 3 with $G(s) = 1/[(s + 2)(s + 10)]$:

- Calculate the unit step responses for $K = 7$ and $K = 20$.
- Verify the steady-state error values of these responses directly.
- Compare the responses on the basis of settling time and nature of the response.

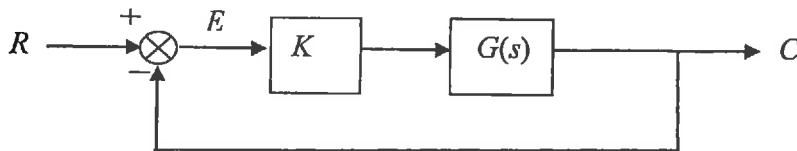


Figure 3

Laplace Transform Table

Laplace Transform $F(s)$	Time function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_c(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n ($n = \text{positive integer}$)
$\frac{1}{s + \alpha}$	$e^{-\alpha t}$
$\frac{1}{(s + \alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s + \alpha)^{n+1}}$	$t^n e^{-\alpha t}$ ($n = \text{positive integer}$)
$\frac{1}{(s + \alpha)(s + \beta)}$	$\frac{1}{\beta - \alpha}(e^{-\alpha t} - e^{-\beta t})$ ($\alpha \neq \beta$)
$\frac{s}{(s + \alpha)(s + \beta)}$	$\frac{1}{\beta - \alpha}(\beta e^{-\alpha t} - \alpha e^{-\beta t})$ ($\alpha \neq \beta$)
$\frac{1}{s(s + \alpha)}$	$\frac{1}{\alpha}(1 - e^{-\alpha t})$
$\frac{1}{s(s + \alpha)^2}$	$\frac{1}{\alpha^2}(1 - e^{-\alpha t} - \alpha t e^{-\alpha t})$
$\frac{1}{s^2(s + \alpha)}$	$\frac{1}{\alpha^2}(\alpha t - 1 + e^{-\alpha t})$
$\frac{1}{s^2(s + \alpha)^2}$	$\frac{1}{\alpha^2}\left[t - \frac{1}{\alpha} + \left(t + \frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s + \alpha)^2}$	$(1 - \alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2 + \omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2 + \omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2 + \omega_n^2)}$	$1 - \cos \omega_n t$
$\frac{\omega_n^2(s + \alpha)}{s^2 + \omega_n^2}$	$\omega_n \sqrt{\alpha^2 + \omega_n^2} \sin(\omega_n t + \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s + \alpha)(s^2 + \omega_n^2)}$	$\frac{\omega_n}{\alpha^2 + \omega_n^2} e^{-\alpha t} + \frac{1}{\sqrt{\alpha^2 + \omega_n^2}} \sin(\omega_n t - \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2} t \quad (\zeta < 1)$
$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$
$\frac{s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t - \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$
$\frac{\omega_n^2(s + \alpha)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\omega_n \sqrt{\frac{\alpha^2 - 2\alpha\zeta\omega_n + \omega_n^2}{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \tan^{-1} \frac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta\omega_n} \quad (\zeta < 1)$
$\frac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n^2 \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1}(2\zeta^2 - 1) \quad (\zeta < 1)$