National Exams May 2016

98-Phys-A7, Optics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate should include in the answer clear statements of the interpretation and any assumptions made.

2. This is a CLOSED BOOK EXAM. Some formulae are listed on pages 7 and 8.

3. Candidates may use one of two calculators, the Casio or Sharp approved models.

4. Answers to questions 1 and 2 plus any two of questions 3 to 6 constitute a complete exam paper.

5. Answer question 1 in the space provided on the exam paper.

6. The first three questions as they appear in the answer book will be marked.

7. Each question is of equal value. Questions 1 and 2 are mandatory.

1. A phrase, or a diagram and a phrase, is all that is required in most cases. [15 marks total, one mark for each letter] **QUESTION 1 IS MANDATORY**. Answer question 1 on these pages, in the space provided.

a) This examination is about light and optics. Define *light*.

b) Define *optics*.

c) Define *geometrical optics*.

d) Define *physical optics*.

e) What problems with *geometrical optics* led to the development of *physical optics*?

f) Define *plane of incidence*.

g) State the *law of reflection*. Include *plane of incidence* in the statement of the law.

h) State the *law of refraction*. Include *plane of incidence* in the statement of the law.

i) Complete the following table:

		wavelength range (nm)	frequency range (Hz)
	UV light		
	red light		
	blue light		
	IR light		

j) Transverse plane waves are solutions to Maxwell's equations for linear, homogeneous, isotropic, stationary media with $\rho_f = 0$ and $J_f = 0$. Define *transverse plane wave*.

k) Is analysis in terms of monochromatic plane waves restrictive? Explain your answer.

l) What is the speed of propagation of the wave $f(x, t) = A\exp(\alpha x)\sin(2\pi(at + bx))$?

m) Define *dispersion* as it relates to refractive index?

n) Define *numerical aperture.*

o) Define *f-number.*

Answer the remaining questions in an answer book.

2. [15 marks total] **QUESTION 2 IS MANDATORY**

a) A collimated beam of light of wavelength 500 nm is normally incident on a slit of width 0.015 cm. A lens of 60 cm focal length is placed behind the slit. Determine, in the focal plane of the lens, (i) the distance between the central maximum and the first minimum, and (ii) the distance between the first and second minima. [4 marks]

b) What is the angular separation between the second-order principal maximum and the neighbouring minima on either side for the diffraction pattern of a 30 groove grating with a groove separation of 0.01 mm and illuminated normally with light of 500 nm. [3 marks]

c) Assume a diffraction limited telescope objective of 10 cm diameter and focal length of 100 cm is used to image light of 500 nm from a distance star. What is the diameter of the Airy disk of the image of the star in the focal plane of the telescope objective? [3 marks]

d) i) State Rayleigh's criterion for resolution and sketch a diagram to demonstrate Rayleigh's criterion. [2 mark]

ii) Assume the pupils of an observer adapt under changes in ambient light from 2 mm to 7 mm diameter. Given the adaptation of the pupils of the observer, over what range of distances can the observer just resolve two lights that are separated laterally by 1 m and that emit at a wavelength of 500 nm. [3 marks]

3. [15 marks total] **ANSWER ANY TWO OF QUESTIONS 3, 4, 5, OR 6.**

a) A ray of light has an angle of incidence of 45 deg and hits the centre of the top surface of a glass cube of refractive index 1.414. Trace the ray through the cube. [3 marks]

b) i) Determine the minimum height of a wall mirror that will permit a 6 ft tall person to view their entire image. ii) Sketch rays from the top and bottom of the person, and determine the proper placement of the mirror such that a full image is observed by the person, regardless of the person's distance from the mirror. [3 marks]

c) A small telescope uses an objective of focal length +12 cm and an eyepiece of focal length − 4 cm. Assume that the telescope is aligned for viewing a near point of 30 cm, i.e., a virtual image that is 30 cm in front of the eye.

i) Determine the separation of the lenses of the telescope. [2 marks]

ii) Trace rays through the telescope for an object that is far away. Use the diagram from the ray tracing to derive the angular magnification of the telescope. [3 marks].

iii) Give the system matrix (as a product of matrices) for the small telescope. [2 marks]

iv) State how one determines the angular magnification and the image distance from the system matrix. [2 marks]

4. [15 marks total] **ANSWER ANY TWO OF QUESTIONS 3, 4, 5, OR 6.**

a) Write equations for the electric field and magnetic fields for TE and TM polarized plane waves. Let the plane of incidence be the x-y plane and allow the waves to be propagating at an angle of θ with respect to the x axis. Assume that the waves propagate in an isotropic, homogeneous medium with refractive index of n. Remember to draw the coordinate system. [6 marks]

b) Check your answers for the **E** and **H** fields and for the vector **k** for part a) by taking limiting cases and by calculating $\mathbf{E}\times\mathbf{H}$. **EXPLAIN** your reasoning. [4 marks]

c) Allow two coherent, monochromatic plane waves of light that are propagating in the x-y plane to interfere. Assume that one plane wave is propagating at an angle of θ with respect to the x axis and the other plane wave is propagating at an angle of $-\theta$ with respect to the x axis. **DERIVE** (i.e., show your work and explain your reasoning) a simplified expression for the time average of the Poynting vector in the plane $x = 0$ that can be used to determine by inspection (hence the request for a simplified expression) the period of the interference fringes in the $x = 0$ plane. [3 marks]

d) Assume that the fringes in part c) are observed in a material with a refractive index $> n$, and that this material has a planar interface starting at the plane $x = 10$. **DERIVE** (i.e., show your work and explain your reasoning) the period of the interference fringes when observed in the material with refractive index $> n$. [2 marks]

5. [15 marks total] **ANSWER ANY TWO OF QUESTIONS 3, 4, 5, OR 6.**

a) Derive the power reflection and transmission coefficients, *R* and *T*, for normal incidence on a planar interface between materials with refractive indices of n_1 and n_2. Draw a sketch of the problem, define a coordinate system, and explain your reasoning for each step in the derivation. [5 marks]

b) Assume air and water with a refractive index of 1.33 Sketch carefully *R* and *T* for both polarizations as a function of the angle of incidence for **internal** reflection. Label the salient features and calculate numerical values for the salient features on both the vertical and horizontal axes. [4 marks]

c) Assume air and water with a refractive index of 1.33 Sketch carefully *R* and *T* for both polarizations as a function of the angle of incidence for **external** reflection. Label the salient features and calculate numerical values for the salient features on both the vertical and horizontal axes. [4 marks]

d) Use your results to explain the conditions wherein polarizing sun glasses are most effective. State the orientation of the transmission axis of the sun glasses under normal conditions of use. [2 marks]

6. [15 marks total] **ANSWER ANY TWO OF QUESTIONS 3, 4, 5, OR 6.**

a) Unpolarized light is passed through three polarizers. The angle between the transmission axes of the first and second polarizer is θ_1 and the angle between the transmission axes of the first and third polarizers is θ_2. What percentage of the unpolarized light is transmitted through the three polarizers? [2 marks]

b) When a polarizer is rotated, the transmitted light shows an intensity variation but there is no angle for the polarizer that gives zero intensity. If a quarter wave plate (QWP) is placed before the polarizer, then rotation of the polarizer can now produce a transmission of zero. What polarizations must be in the incident light? Explain. [3 marks]

c) A monochromatic beam, of wavelength λ, of linearly polarized light is converted to circularly polarized light by transmission through a birefringent material of thickness *t*. Assume that *t* is the minimum thickness for which the conversion is observed.

i) Sketch front and side views of the optical system. Choose a coordinate system and indicate the coordinate system on the sketch. Indicate directions and variables of interest on the sketches. [3 marks]

ii) Calculate the difference in refractive indices for the two orthogonal polarizations. Provide calculations to support your answer. [4 marks]

iii) Provide equations for the electric field of the light entering the material and for the electric field of the light exiting the material. [3 marks]

Question 6 c) iii) is the last question. Some formulae follow.

$$E(x,y,z) = \frac{ik}{2\pi}\frac{e^{ikz}}{z} e^{i\frac{k}{2z}(x^2+y^2)} \iint E(x_a, y_a, 0) e^{i\frac{k}{2z}(x_a^2+y_a^2)} e^{-i\frac{k}{z}(x\,x_a + y\,y_a)}\, \mathrm{d}x_a\, \mathrm{d}y_a$$

The field in the neighbourhood of the focus of a circular lens of radius a is given in the usual paraxial approximation as

$$E(u,v) = \int_0^1 \mathrm{J}_0(2\pi v \rho_a)\, e^{-i\pi u \rho_a^2} \rho_a\, \mathrm{d}\rho_a$$

with $\rho_a = \sqrt{(x_a^2 + y_a^2)}$, $\quad u = \frac{1}{\lambda}\frac{a^2}{q(q+\Delta)}\Delta$, $\quad v = \frac{1}{\lambda}\frac{a\sqrt{(x^2+y^2)}}{(q+\Delta)}$.

$\mathrm{J}_0(0) = 1$; $\mathrm{J}_0(2.4048) = 0$; $\mathrm{J}_0(5.5201) = 0$; $\mathrm{J}_0(8.6537) = 0$; $\mathrm{J}_0(11.7915) = 0$;

$\gamma = ½\, k\, D \sin(\theta)$. The zeros for $J_1(\gamma)$ occur for γ = 0, 3.832, 5.136, 7.016, 8.417, 10.173, 11.620, 13.324, . . .

$$\frac{d}{dx} x^n J_n(x) = x^n J_{n-1}(x)$$

$$(1 + \epsilon)^{\xi} = 1 + \frac{\xi}{1!}\epsilon + \frac{\xi\times(\xi-1)}{2!}\epsilon^2 + \ldots$$

zone plate radii $R_m = (m\, r_o\, \lambda)^{0.5}$

The intensity in the far-field as a function of the angle θ_m from the normal of a diffraction grating of N lines, line spacing of a, and line width of b, for illumination with a plane wave with $k = 2\pi/\lambda$ and an angle of incidence of θ_i is

$$I(\theta) = I_o\left[\frac{\sin(\beta)}{\beta}\right]^2\left[\frac{\sin(N\alpha)}{\sin(\alpha)}\right]^2$$

$$\beta = \frac{k\,b}{2}\left(\sin(\theta_i) + \sin(\theta_m)\right)$$

$$\alpha = \frac{k\,a}{2}\left(\sin(\theta_i) + \sin(\theta_m)\right)$$

For a blazed grating, $2\theta_b = \theta_i - \theta_m$

The resolution R and the dispersion D for a grating with N lines and order m are

$$R = \frac{\lambda}{\Delta\lambda} = mN \qquad\qquad D = \frac{m}{a\cos(\theta)} .$$

double angle formulae:

$$
\begin{aligned}
\sin(A+B) &= \sin(A)\sin(B) + \cos(A)\cos(B) \\
\sin(A-B) &= \sin(A)\sin(B) - \cos(A)\cos(B) \\
\cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B) \\
\cos(A-B) &= \cos(A)\cos(B) + \sin(A)\sin(B)
\end{aligned}
$$

$$
\begin{aligned}
\sin(A) + \sin(B) &= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\
\sin(A) - \sin(B) &= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\
\cos(A) + \cos(B) &= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\
\cos(A) - \cos(B) &= 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)
\end{aligned}
$$

$$
v_p = \frac{\omega}{k} \qquad v_g = \frac{d\omega}{dk}
$$

The translation, refraction at a spherical interface, thin lens, and spherical mirror matrices are listed below.

$$
\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \quad
\begin{bmatrix} 1 & 0 \\ \dfrac{n_1 - n_2}{R n_2} & \dfrac{n_1}{n_2} \end{bmatrix} \quad
\begin{bmatrix} 1 & 0 \\ \dfrac{-1}{f} & 1 \end{bmatrix} \quad
\begin{bmatrix} 1 & 0 \\ \dfrac{-2}{R} & 1 \end{bmatrix}
$$

THE END